

Matematik 5 svar

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Kapitel 4

4101. a) $y = (2x + 3)^4 \Rightarrow y'(x) = 4(2x + 3)^3 \cdot 2 = 8(2x + 3)^3$

b) $y = (1 - 4x)^5 \Rightarrow y'(x) = -5 \cdot 4(1 - 4x)^4 = -20(1 - 4x)^4$

c) $y = (1 + 2x^3)^4 \Rightarrow y'(x) = 4 \cdot 2 \cdot 3x^2(1 + 2x^3)^3 = 24x^2(1 + 2x^3)^3$

d) $y = (x^2 + 2x)^3 \Rightarrow y'(x) = 3(x^2 + 2x)^2(2x + 2) = 6(x + 1)(x^2 + 2x)^2$

e) $y = \left(1 + \frac{1}{2x^3}\right)^4 = \left(1 + \frac{1}{2}x^{-3}\right)^4 \Rightarrow y'(x) = 4\left(1 + \frac{1}{2x^3}\right)^3 \cdot \frac{-3}{2}x^{-4} = -\frac{6}{x^4}\left(1 + \frac{1}{2x^3}\right)^3$

f) $y = (\sin x + \cos x)^3 \Rightarrow y'(x) = 3(\cos x - \sin x)(\sin x + \cos x)^2$

4102. a) $f(x) = (3x + 5)^{20} \Rightarrow f'(x) = 20(3x + 5)^{19} \cdot 3 = 60x(3x + 5)^{19}$

b) $g(x) = \sqrt{3x - 5} \Rightarrow g'(x) = \frac{3}{2\sqrt{3x-5}}$

c) $h(x) = \frac{1}{(2x+1)^2} = (2x+1)^{-2} \Rightarrow h'(x) = -2 \cdot 2(2x+1)^{-3} = -\frac{4}{(2x+1)^3}$

d) $f(t) = (e^{3t} + t)^4 \Rightarrow f'(t) = 4(3e^{3t} + 1)(e^{3t} + t)^3$

e) $g(u) = \frac{2}{e^{3u}} = 2e^{-3u} \Rightarrow g'(u) = -6e^{-3u} = -\frac{6}{e^{3u}}$

f) $h(v) = \frac{1}{\sqrt{v^2+3}} = (v^2 + 3)^{-\frac{1}{2}} \Rightarrow h'(v) = -\frac{1}{2}2v(v^2 + 3)^{-\frac{3}{2}} = -v(v^2 + 3)^{-\frac{3}{2}} = \frac{-v}{(v^2+3)^{1.5}}$

4103. a) $f(x) = e^{x^2} \Rightarrow f'(x) = 2xe^{x^2}$

b) $f(v) = \sin^2 v \Rightarrow f'(v) = 2 \sin v \cos v = \sin 2v$

4104. a) $y = (\sin x + x)^2 \Rightarrow y'(x) = 2(\sin x + x)(\cos x + 1)$

b) $y = (2x + e^{2x})^4 \Rightarrow y'(x) = 4(2x + e^{2x})^3(2 + 2e^{2x})$

c) $y = e^{(x^2+1)^2} \Rightarrow y'(x) = 2(x^2 + 1)2xe^{(x^2+1)^2} = 4x(x^2 + 1)e^{(x^2+1)^2}$

d)

$$y = \ln(x + 2x^3) \Rightarrow y'(x) = \frac{1 + 6x^2}{x + 2x^3}$$

4105. A: $f(x) = \ln e^{3x} = 3x \ln e = 3x \Rightarrow f'(x) = 3$

B:

$$f(x) = \ln(\cos 3x) \Rightarrow f'(x) = -\frac{3 \sin 3x}{\cos 3x} = -3 \tan 3x$$

4106. a)

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = 5x^2$$

b)

$$\frac{dz}{dx} = \frac{dy}{dx} \cdot \frac{dz}{dy} = \frac{dy}{dx} \cdot \left(\frac{dy}{dz}\right)^{-1} = 8 \cdot \frac{1}{2} = 4$$

4107. a) $y = x^3(x^2 - 1)^3 \Rightarrow y' = 3x^2(x^2 - 1)^3 + x^3 \cdot 3(x^2 - 1)^2 \cdot 2x =$

$$= 3x^2(x^2 - 1)^3 + 6x^4(x^2 - 1)^2 = 3x^2(x^2 - 1)^2(3x^2 - 1)$$

b) $y = (1 + x^2)(x^2 - 1)^3 \Rightarrow y' = 2x(x^2 - 1)^3 + (1 + x^2) \cdot 3 \cdot 2x(x^2 - 1)^2 =$

$$= 2x(x^2 - 1)^2[x^2 - 1 + (1 + x^2)3] = 2x(x^2 - 1)^2[x^2 - 1 + 3 + 3x^2] =$$

$$= 4x(x^2 - 1)^2(2x^2 + 1)$$

c) $= e^{x^2}(x^3 - 1)^2 \Rightarrow y' = 2xe^{x^2}(x^3 - 1)^2 + e^{x^2} \cdot 2 \cdot 3x^2(x^3 - 1) =$

$$= 2xe^{x^2}(x^3 - 1)(x^3 - 1 + 3x)$$

d) $y = \sin 2x(\cos 4x + 2) \Rightarrow y' = 2\cos 2x(\cos 4x + 2) - 4\sin 2x \sin 4x =$

$$= 2\cos 2x(\cos 4x + 2) - 4\sin 2x \cdot 2\sin 2x \cos 2x = 2\cos 2x(\cos 4x + 2 - 4\sin^2 2x)$$

4108. $\frac{ds}{dt} = 0.3 \frac{\text{cm}}{\text{h}}, V = s^3 \Rightarrow \frac{dV}{ds} = 3s^2$ och $\frac{dV}{dt} = \frac{dV}{ds} \frac{ds}{dt} = \frac{dV}{ds} \frac{ds}{dt} = 3s^2 \cdot 0.3 \approx 5.1 \text{ dm}^3/\text{h}$

4109. $A = x^2, \frac{dA}{dt} = 2x \frac{ds}{ds} = 2 \cdot 15 \cdot 3.7 = 111 \frac{\text{cm}^2}{\text{s}} \approx 1.1 \text{ dm}^2/\text{s}$

4110. $50 \text{ m}^2 \cdot \frac{dx}{dt} = 0.09 \frac{\text{m}^3}{\text{min}} \Rightarrow \frac{dx}{dt} = \frac{0.09 \text{ m}^3}{50 \text{ min m}^2} = 1.8 \frac{\text{mm}}{\text{min}}$

4111. $V = A \cdot h \Rightarrow \frac{dV}{dt} = A \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{A} \frac{dV}{dt} = \frac{1}{\pi \cdot 1.2^2} \cdot 0.065 \frac{\text{m}}{\text{min}} \approx 1.4 \text{ cm}/\text{min}$

4112. $V = \frac{4\pi r^3}{3}, \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2} \frac{dV}{dt} = \frac{1}{4\pi \cdot 25^2} \cdot 1600 \approx 2 \text{ mm}/\text{s}$

4113. $\frac{h}{r} = 1.5, \frac{dh}{dt} = \frac{dh}{dt} \frac{dV}{dV} = \frac{dV}{dt} \frac{dh}{dV} = \frac{dV}{dt} \left(\frac{dV}{dh}\right)^{-1}$ men $V = \frac{bh}{3} = \frac{\pi r^2 h}{3} = \frac{\pi \left(\frac{h}{1.5}\right)^2 h}{3} = \frac{\pi h^3}{6.75}$

$$\frac{dV}{dh} = \frac{\pi 3h^2}{6.75} = \frac{\pi h^2}{2.25} \Rightarrow \frac{dh}{dt} = \frac{dV}{dt} \left(\frac{dV}{dh} \right)^{-1} = 2 \cdot \frac{2.25}{\pi 2^2} \approx 3.6 \text{ cm/min}$$

$$4114. A = \pi r^2, \frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dr} \frac{dr}{dt} = \pi 2r \cdot \frac{dr}{dt} = \pi 2 \cdot (4 \cdot 1.2) \cdot 1.2 \approx 36 \text{ dm}^2/\text{s}$$

4115.

$$\frac{1}{2} = \frac{h}{d} = \frac{h}{2r} \Rightarrow h = r$$

$$\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} = \frac{dV}{dt} \left(\frac{dV}{dh} \right)^{-1} \text{ men } V = \frac{bh}{3} = \frac{\pi r^2 h}{3} = \frac{\pi h^2 h}{3} = \frac{\pi h^3}{3}$$

$$\frac{dV}{dh} = \frac{\pi 3h^2}{3} = \pi h^2 \Rightarrow \frac{dh}{dt} = 0.4 \cdot \frac{1}{\pi 2^2} = \frac{0.1}{\pi} \approx 3.2 \text{ cm/min}$$

4116. a)

$$\frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{dV}{dt} \left(\frac{dV}{dr} \right)^{-1} = \frac{dV}{dt} \left(\frac{d}{dr} \frac{4\pi r^3}{3} \right)^{-1} \Rightarrow$$

$$\frac{dr}{dt} = \frac{dV}{dt} (4\pi r^2)^{-1} = \frac{150}{4\pi 25^2} \approx 0.19 \text{ mm/min}$$

b)

$$\frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt} = \frac{dr}{dt} \left(\frac{d}{dr} 4\pi r^2 \right) = \frac{150}{4\pi 25^2} \cdot 8\pi 25 = 12 \text{ cm}^2/\text{min}$$

4117. Konens volym finner man som:

$$V_{\text{kon}} = \frac{bh}{3} = \frac{\pi r^2 h}{3} = \{\text{toppvinkel} = 90^\circ \Rightarrow r = h\} = \frac{\pi h^3}{3}$$

$$\frac{dV}{dh} = \pi h^2 \text{ dvs } \frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \left(\frac{dV}{dh} \right)^{-1} \frac{dV}{dt} = \frac{1}{\pi 3^2} 2.5 \approx 8.8 \text{ mm/min}$$

4118.

$$V = \frac{4\pi r^3}{3}, A = 4\pi r^2, \frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} \text{ är konstant}$$

$$4119. y^2 = 2x \{\text{implicit derivering ger}\} 2y \frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y} \Rightarrow y'(2) = -\frac{1}{2}$$

$$4120. \text{ a) } y^2 = 9x \{\text{implicit derivering ger}\} 2y \frac{dy}{dx} = 9 \Rightarrow \frac{dy}{dx} = \frac{9}{2y} \Rightarrow y'(4) = \frac{9}{12} = \frac{3}{4}$$

$$\text{ b) Enpunktsformeln ger } y - 6 = \frac{3}{4}(x - 4) \Rightarrow y = \frac{3}{4}x + 6 - 4 \frac{3}{4} = \frac{3}{4}x + 3$$

$$\text{ c) } y = -\frac{3}{4}x - 3 \text{ (fel i facit)}$$

$$4121. \text{ a) } x^2 + 2y^2 = 1 \{\text{implicit derivering ger}\} 2x + 4y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{2y}$$

$$\text{ b) } x^2 + 2y = y^3 \{\text{implicit derivering ger}\} 2x + 2 \frac{dy}{dx} = 3y^2 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{2x}{3y^2 - 2}$$

$$4122. x^2 + y^2 = 25 \text{ \{implicit derivering ger\}} 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

Lutningen i (3,4) är $-3/4$.

$$4123. 2x^2 + 3y^2 = 12 \text{ \{implicit derivering ger\}} 4x + 6y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{3y}$$

$$4124. 2x^2 + 3y^2 = 14 \text{ \{implicit derivering ger\}} 4x + 6y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{3y} = -\frac{1}{3}$$

$$4125. x^2 + y^2 = 2 \text{ \{implicit derivering ger\}} 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

$$\text{Enpunktsformeln ger: } y - (-1) = -\frac{-1}{-1}(x - (-1)) \Rightarrow y = -x - 2$$

4127.

$$\frac{d}{dx}(x^2 \cdot y) = 0 \Rightarrow 2x \cdot y + x^2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2xy}{x^2} = -\frac{2}{x} \cdot y = -\frac{2}{x^3}$$

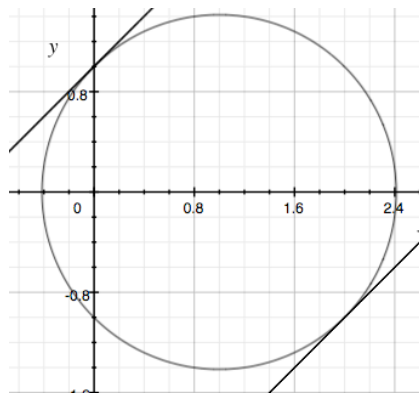
$$y = \frac{1}{x^2} \Rightarrow y' = -\frac{2}{x^3}$$

Ja, samma resultat.

4128.

$$(x-1)^2 + y^2 = 2 \quad \frac{d}{dx} \Rightarrow 2(x-1) + 2y \frac{dy}{dx} = 0 \text{ dvs då } 1-x=y$$

$$y^2 + y^2 = 2 \Rightarrow (x_1, y_1) = (0,1) \text{ och } (x_2, y_2) = (2,-1)$$



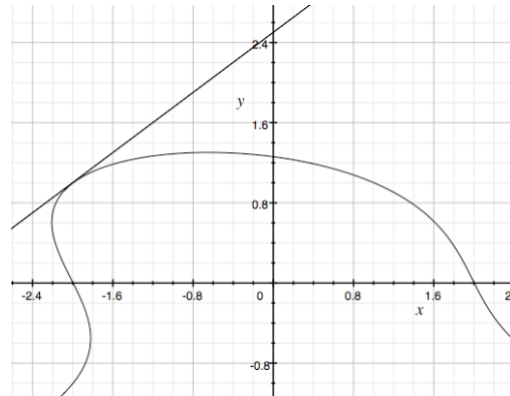
4129.

$$x^2 + xy + 2y^3 = 4 \quad \frac{d}{dx} \Rightarrow 2x + y + x \frac{dy}{dx} + 2 \cdot 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 6y^2) = -2x - y \Rightarrow \frac{dy}{dx} = -\frac{2x + y}{x + 6y^2} = -\frac{-4 + 1}{-2 + 6} = \frac{3}{4}$$

$$y = kx + m \Rightarrow 1 = \frac{3}{4}(-2) + m \Rightarrow m = 1 + \frac{3}{2} = 2.5$$

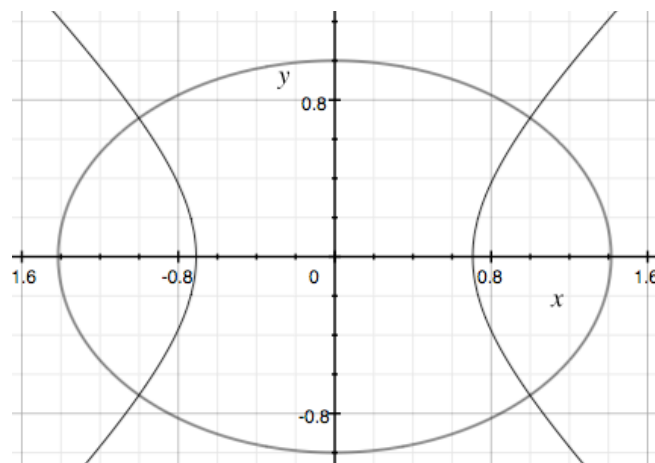
$$y = 0.75x + 2.5$$



4130.

$$\begin{cases} x^2 + 2y^2 = 2 \\ 2x^2 - 2y^2 = 1 \end{cases} \Rightarrow 3x^2 = 3 \Rightarrow \begin{cases} x = \pm 1 \\ y = \pm \frac{1}{\sqrt{2}} \end{cases}$$

$$\frac{d}{dx} \begin{cases} x^2 + 2y^2 = 2 \\ 2x^2 - 2y^2 = 1 \end{cases} \Rightarrow \begin{cases} 2x + 4y \frac{dy}{dx} = 0 \\ 4x - 4y \frac{dy}{dx} = 0 \end{cases} \Rightarrow \begin{cases} \frac{dy}{dx} = -\frac{x}{2y} = -\frac{1}{\sqrt{2}} \\ \frac{dy}{dx} = \frac{x}{y} = \frac{1}{1/\sqrt{2}} = \sqrt{2} \end{cases} \text{ VSV}$$



4131.

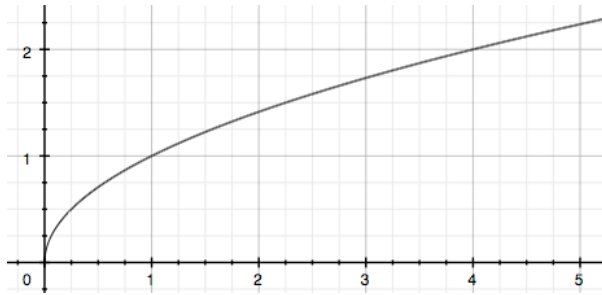
$$\frac{d}{dt}(y^2 = 300^2 + x^2) \Rightarrow 2y \frac{dy}{dt} = 2x \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{x}{\sqrt{300^2 + x^2}} 50 \text{ m/min}$$

a) $\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{300}{\sqrt{300^2 + 300^2}} 50 \approx 35 \text{ m/min}$

b) $\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{400}{\sqrt{300^2 + 400^2}} 50 = 40 \text{ m/min}$

4132.

$$\frac{d}{dt}(y^2 = x) \Rightarrow 2y \frac{dy}{dt} = \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{1}{2y} \frac{dx}{dt} = \frac{1.5}{4} = \frac{3}{8} \approx 0.38 \text{ m/s}$$



4133.

$$x^2 = 1500^2 + y^2, \quad \frac{d}{dt}(x^2 = 1500^2 + y^2) \Rightarrow 2x \frac{dx}{dt} = 2y \frac{dy}{dt} \Rightarrow$$

$$\frac{dx}{dt} = \frac{y}{x} \frac{dy}{dt} = \frac{1000}{\sqrt{1500^2 + 1000^2}} 200 \approx 110 \text{ m/s}$$

4134.

$$\frac{d}{dt}(y^2 = 5^2 - x^2) \Rightarrow 2y \frac{dy}{dt} = -2x \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = -\frac{3}{\sqrt{5^2 - 3^2}} 0.04 = -0.03 \text{ m/s}$$

4135.

$$v = 540 \frac{\text{km}}{\text{h}} = 150 \text{ m/s}$$

$$a(t) = \sqrt{(vt)^2 + h^2} \Rightarrow \frac{da}{dt} = \frac{1}{2} \frac{2tv^2}{\sqrt{(vt)^2 + h^2}} = \{t = 60 \text{ s}\} \approx 130 \frac{\text{m}}{\text{s}} \approx 470 \text{ km/h}$$

4201. a)

$$\int_0^{\infty} e^{-x} dx = \lim_{t \rightarrow \infty} \int_0^t e^{-x} dx = \lim_{t \rightarrow \infty} [e^{-x}]_0^t = 1$$

b)

$$\int_0^{\infty} e^{-2x} dx = \lim_{t \rightarrow \infty} \int_0^t e^{-2x} dx = \frac{1}{2} \lim_{t \rightarrow \infty} [e^{-2x}]_0^t = \frac{1}{2}$$

c)

$$\int_0^{\infty} \sin x dx = \lim_{t \rightarrow \infty} \int_0^t \sin x dx = \lim_{t \rightarrow \infty} [\cos x]_0^t \Rightarrow \text{divergent}$$

4202. a)

$$\int_1^{\infty} \frac{1}{\sqrt{x}} dx = [2\sqrt{x}]_1^{\infty} \Rightarrow \text{divergent}$$

b)

$$\int_1^{\infty} \frac{1}{x} dx = [\ln x]_1^{\infty} \Rightarrow \text{divergent}$$

c)

$$\int_1^{\infty} \frac{1}{x^2} dx = \left[\frac{1}{x}\right]_1^{\infty} = 1$$

4203.

$$A = \int_0^{\infty} e^{-0.2x} dx = [5e^{-0.2x}]_0^{\infty} = 5 \text{ a. e.}$$

4204. a)

$$\lim_{w \rightarrow \infty} \int_2^w \frac{2}{x^5} dx = \lim_{w \rightarrow \infty} \left[-\frac{2}{4x^4}\right]_2^w = \frac{1}{2} \lim_{w \rightarrow \infty} \left[\frac{1}{2^4} - \frac{1}{w^4}\right] = \frac{1}{32}$$

b)

$$\lim_{w \rightarrow \infty} \int_0^w \frac{4}{(2x+1)^3} dx = \lim_{w \rightarrow \infty} \left[-\frac{1}{2} \cdot \frac{1}{2} \frac{4}{(2x+1)^2}\right]_0^w = \lim_{w \rightarrow \infty} \left[1 - \frac{1}{(2w+1)^2}\right] = 1$$

c)

$$\lim_{w \rightarrow \infty} \int_0^w 3e^{-2x} dx = \lim_{w \rightarrow \infty} \left[-\frac{3}{2} e^{-2x}\right]_0^w = \frac{3}{2}$$

d)

$$\lim_{w \rightarrow \infty} \int_4^w \frac{2}{x\sqrt{x}} dx = \lim_{w \rightarrow \infty} \left[-2 \frac{2}{\sqrt{x}}\right]_4^w = 2$$

e)

$$\lim_{w \rightarrow \infty} \int_1^w \frac{2}{\sqrt{x}} dx = \lim_{w \rightarrow \infty} [2 \cdot 2\sqrt{x}]_1^w \text{ divergerar}$$

f)

$$\lim_{w \rightarrow \infty} \int_{-w}^1 e^{2x} dx = \lim_{w \rightarrow \infty} \left[\frac{1}{2} e^{2x}\right]_{-w}^1 = \frac{e^2}{2}$$

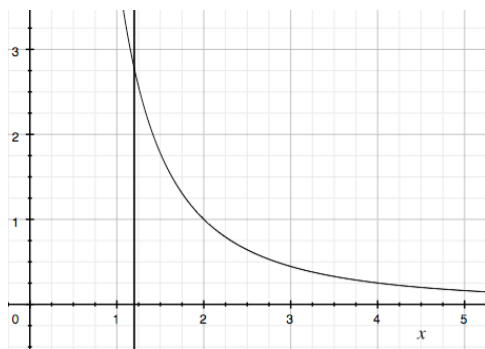
4205.

$$\lim_{a \rightarrow \infty} \int_0^a 4e^{-0.9x} dx = \lim_{a \rightarrow \infty} \left[-\frac{4}{0.9} e^{-0.9x}\right]_0^a = \frac{4}{0.9}$$

4206.

$$\int_0^1 \frac{1}{\sqrt{x}} dx = [2\sqrt{x}]_0^1 = 2$$

4207.



$$\lim_{w \rightarrow \infty} \int_a^w \frac{4}{x^2} dx = \lim_{w \rightarrow \infty} \left[-\frac{4}{x} \right]_a^w = \frac{4}{a} = 4 \Rightarrow a = 1$$

4208.

$$\begin{aligned} \lim_{w \rightarrow \infty} \int_1^w \frac{1}{x^k} dx &= \lim_{w \rightarrow \infty} \int_1^w x^{-k} dx = \lim_{w \rightarrow \infty} \left[\frac{1}{-k+1} x^{-k+1} \right]_1^w = \frac{1}{1-k} \lim_{w \rightarrow \infty} \left[\frac{1}{x^{k-1}} \right]_1^w \\ &= \frac{1}{1-k} \lim_{w \rightarrow \infty} \left(\frac{1}{w^{k-1}} - 1 \right) \text{ gränsvärdet konvergerar om } k > 1. \end{aligned}$$

4209.

$$V = \int_1^5 \pi(2x)^2 dx = 4\pi \int_1^5 x^2 dx = \frac{4\pi}{3} [x^3]_1^5 = \frac{4\pi 124}{3} \approx 519 \text{ v. e.}$$

Volymen kallas en stympad kon.

4212.

$$V = \lim_{w \rightarrow \infty} \int_0^w \pi e^{-2x} dx = \frac{\pi}{2} \lim_{w \rightarrow \infty} [e^{-2x}]_0^w = \frac{\pi}{2} \text{ v. e.}$$

4213. Volymen är den som i vänsterkant begränsas av $x = 2$.

$$V = \lim_{w \rightarrow \infty} \int_2^w \pi \left(\frac{1}{x\sqrt{x}} \right)^2 dx = \pi \lim_{w \rightarrow \infty} \int_2^w \frac{1}{x^3} dx = \frac{\pi}{2} \lim_{w \rightarrow \infty} \left[\frac{1}{x^2} \right]_2^w = \frac{\pi}{8} \text{ v. e.}$$

4214. Volymen är den som i vänsterkant begränsas av $x = 2$.

$$V = \lim_{w \rightarrow \infty} \int_2^w \frac{\pi dx}{(x-1)^2} = \pi \lim_{w \rightarrow \infty} \left[\frac{1}{x-1} \right]_2^w = \pi \text{ v. e.}$$

4215. a)

$$\lim_{w \rightarrow \infty} \int_1^w \frac{1}{x} dx = \lim_{w \rightarrow \infty} [\ln|x|]_1^w \text{ divergent} \Rightarrow \text{oänligt mycket färg!}$$

b)

$$\lim_{w \rightarrow \infty} \int_1^w \pi \frac{1}{x^2} dx = \pi \lim_{w \rightarrow \infty} \left[\frac{1}{x} \right]_1^w = \pi \text{ v. e. färg.}$$

4216.

$$\int_{-1}^3 \pi(x+1) dx = \pi \left[\frac{x^2}{2} + x \right]_{-1}^3 = \pi \left(\frac{9}{2} + 3 - \frac{1}{2} + 1 \right) = 8\pi \text{ v. e.}$$

4217.

$$\int_{-1}^a \pi(x+1) dx = \pi \left[\frac{x^2}{2} + x \right]_{-1}^a = \pi \left(\frac{a^2}{2} + a - \frac{1}{2} + 1 \right) = 16\pi$$
$$\frac{a^2}{2} + a - \frac{1}{2} + 1 = 16 \Rightarrow a^2 + 2a - 31 = 0$$
$$a = -1 \pm \sqrt{1+31} - 1 \pm \sqrt{32} = 4\sqrt{2} - 1 \approx 4.7$$

4218. a)

$$\int_0^2 10e^{-0.5t} dt = 10 \cdot 2 [e^{-0.5t}]_2^0 = 20(1 - e^{-1}) \approx 12.6 \text{ g}$$

b) 20 g

4219.

$$\lim_{w \rightarrow \infty} \int_0^w 100e^{-0.05t} dt = 2000 \lim_{w \rightarrow \infty} [e^{-0.05t}]_w^0 = 2000 \text{ liter}$$

4220.

$$\lim_{w \rightarrow \infty} \int_0^w 2e^{-0.025x} dx = 80 \lim_{w \rightarrow \infty} [e^{-0.025t}]_w^0 = 80$$

Den ursprungliga temperaturen var 80° .

4221.

$$\lim_{w \rightarrow \infty} \int_0^w 0.001e^{-0.001x} dx = \lim_{w \rightarrow \infty} [e^{-0.001t}]_w^0 = 1$$

Sannolikheten att den förr eller senare går sönder = 1.

4222. a)

$$5200 \int_0^2 \frac{dt}{(1+t)^2} = 5200 \left[\frac{1}{1+t} \right]_2^0 = 5200 \left(1 - \frac{1}{3} \right) \approx 3500 \text{ st}$$

b) 5200 st

4223.

$$\lim_{w \rightarrow \infty} \int_0^w 0.001 e^{-0.05t} dt = 0.02 \lim_{w \rightarrow \infty} [e^{-0.05t}]_0^w = 0.02 \text{ g}$$

Facit svarar μg , troligen skulle det varit μg i uppgiften.

4224.

$$\begin{aligned} i(t) &= 4.41 \cdot 10^{-5} \cdot 0.956^t = 4.41 \cdot 10^{-5} \cdot e^{t \ln 0.956} \\ q_{tot} &= 4.41 \cdot 10^{-5} \lim_{w \rightarrow \infty} \int_0^w e^{t \ln 0.956} dt = \frac{4.41 \cdot 10^{-5}}{\ln 0.956} \lim_{w \rightarrow \infty} [e^{t \ln 0.956}]_0^w = \\ &= \frac{4.41 \cdot 10^{-5}}{\ln 0.956} (-1) = 0.98 \text{ mC} \end{aligned}$$

4225. a)

$$\begin{aligned} \int_{R_j}^R F(r) dr &= 6.672 \cdot 10^{-11} M \cdot m \int_{R_j}^R \frac{dr}{r^2} = 6.672 \cdot 10^{-11} M \cdot m \left[\frac{1}{r} \right]_{R_j}^R = \\ &= 6.672 \cdot 10^{-11} M \cdot m \left(\frac{1}{R_j} - \frac{1}{R} \right) = \\ &= 6.672 \cdot 10^{-11} \cdot 5.9735 \cdot 10^{24} \cdot 15\,000 \left(\frac{1}{6.37 \cdot 10^6} - \frac{1}{6.38 \cdot 10^6} \right) \approx 1.47 \text{ GJ} \end{aligned}$$

Energin som krävs att lyfta 15 ton 10 km ut från jordens yta.

b) Cirka 940 GJ för att lyfta 15 ton bort från jordens gravitationsfält.

$$4301. \text{ a) } \frac{x^2+3}{x^2} = 1 + \frac{3}{x^2} \quad \text{b) } \frac{10-x}{2x} = \frac{10}{2x} - \frac{x}{2x} = \frac{5}{x} - \frac{1}{2} \quad \text{c) } \frac{x(x+2)+1}{x+2} = \frac{x(x+2)}{x+2} + \frac{1}{x+2} = x + \frac{1}{x+2}$$

$$4302. \text{ a) } \frac{x}{x+4} = \frac{x+4-4}{x+4} = \frac{x+4}{x+4} - \frac{4}{x+4} = 1 - \frac{4}{x+4} \quad \text{b) } \frac{x}{x-7} = \frac{x-7+7}{x-7} = \frac{x-7}{x-7} + \frac{7}{x-7} = 1 + \frac{7}{x-7}$$

$$4303. \text{ a) } \frac{x}{x+1} = \frac{x+1-1}{x+1} = \frac{x+1}{x+1} - \frac{1}{x+1} = 1 - \frac{1}{x+1} \quad \text{b) } \frac{x}{x-5} = \frac{x-5+5}{x-5} = \frac{x-5}{x-5} + \frac{5}{x-5} = 1 + \frac{5}{x-5}$$

$$\text{c) } \frac{x}{x+8} = \frac{x+8-8}{x+8} = \frac{x+8}{x+8} - \frac{8}{x+8} = 1 - \frac{8}{x+8}$$

$$4304. \frac{1}{t(t+1)} = \frac{a}{t} + \frac{b}{t+1} = \frac{a(t+1)}{t(t+1)} + \frac{bt}{(t+1)t} = \frac{at+a+bt}{t(t+1)} = \begin{cases} a = 1 \\ b = -1 \end{cases} = \frac{1}{t} - \frac{1}{t+1}$$

4305.

$$\begin{aligned} \frac{3x}{(x+2)(x-1)} &= \frac{a}{x+2} + \frac{b}{x-1} = \frac{a(x-1)}{(x+2)(x-1)} + \frac{b(x+2)}{(x+2)(x-1)} = \\ &= \frac{a(x-1) + b(x+2)}{(x+2)(x-1)} = \begin{cases} a+b=3 \\ -a+2b=0 \end{cases} \Rightarrow \begin{cases} b=1 \\ a=2 \end{cases} = \frac{2}{x+2} + \frac{1}{x-1} \end{aligned}$$

4306. a)

$$\frac{x+1}{x+2} = \frac{x+1+1-1}{x+2} = \frac{x+2-1}{x+2} = \frac{x+2}{x+2} - \frac{1}{x+2} = 1 - \frac{1}{x+2}$$

b)

$$\frac{x-2}{x+1} = \frac{x-2+3-3}{x+1} = \frac{x+1-3}{x+1} = \frac{x+1}{x+1} - \frac{3}{x+1} = 1 - \frac{3}{x+1}$$

c)

$$\frac{x+1}{x-3} = \frac{x+1-4+4}{x-3} = \frac{x-3}{x-3} + \frac{4}{x-3} = 1 + \frac{4}{x-3}$$

4307. a)

$$\frac{x+1}{(x-2)(x-1)} = \frac{a}{x-2} + \frac{b}{x-1} = \frac{a(x-1)}{(x-2)(x-1)} + \frac{b(x-2)}{(x-2)(x-1)} =$$

$$= \frac{a(x-1) + b(x-2)}{(x-2)(x-1)} = \begin{cases} a+b=1 \\ -a-2b=1 \end{cases} \Rightarrow \begin{cases} b=-2 \\ a=3 \end{cases} =$$

$$= \frac{3}{x-2} - \frac{2}{x-1}$$

b)

$$\frac{4}{x(x-4)} = \frac{a}{x} + \frac{b}{x-4} = \frac{a(x-4)}{x(x-4)} + \frac{bx}{x(x-4)} = \frac{a(x-4) + bx}{x(x-4)} =$$

$$= \begin{cases} a=-1 \\ b=1 \end{cases} = \frac{1}{x-4} - \frac{1}{x}$$

c)

$$\frac{2x+5}{x^2-5x} = \frac{2x+5}{x(x-5)} = \frac{a}{x} + \frac{b}{x-5} = \frac{a(x-5) + bx}{x(x-5)} = \begin{cases} a+b=2 \\ a=-1 \\ b=3 \end{cases} =$$

$$= \frac{3}{x-5} - \frac{1}{x}$$

4308.

$$\frac{1-x}{(x+2)^2} = \frac{a}{x+2} + \frac{b}{(x+2)^2} = \frac{a(x+2) + b}{(x+2)^2} = \begin{cases} a=-1 \\ b=3 \end{cases} = \frac{3}{(x+2)^2} - \frac{1}{x+2}$$

4309. a)

$$\frac{x}{(x-1)^2} = \frac{a}{(x-1)^2} + \frac{b}{x-1} = \frac{a + b(x-1)}{(x-1)^2} = \begin{cases} b=1 \\ a=1 \end{cases} = \frac{1}{(x-1)^2} + \frac{1}{x-1}$$

b)

$$\frac{x+3}{(x-1)^2} = \frac{a}{(x-1)^2} + \frac{b}{x-1} = \frac{a + b(x-1)}{(x-1)^2} = \begin{cases} b=1 \\ a=4 \end{cases} = \frac{4}{(x-1)^2} + \frac{1}{x-1}$$

4310. a)

$$\int_1^3 \frac{3x+x^2}{x} dx = \int_1^3 (3+x) dx = \left[3x + \frac{x^2}{2} \right]_1^3 = 9 + \frac{9}{2} - 3 - \frac{1}{2} = 10$$

b)

$$\int_1^2 \frac{2+x}{x} dx = \int_1^2 \left(\frac{2}{x} + 1 \right) dx = [2 \ln x + x]_1^2 = 2 \ln 2 + 2 - 2 \ln 1 - 1 = 2 \ln 2 + 1$$

4311. a)

$$\int \frac{x}{x+2} dx = \int \frac{x+2}{x+2} - \frac{2}{x+2} dx = x - 2 \ln|x+2| + C$$

b)

$$\int \frac{x}{x-2} dx = \int \frac{x-2}{x-2} + \frac{2}{x-2} dx = x + 2 \ln|x-2| + C$$

4312. a)

$$\begin{aligned} \int_2^3 \frac{1}{x(x+1)} dx &= \left[\frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1) + Bx}{x(x+1)} \Rightarrow \begin{matrix} A = 1 \\ B = -1 \end{matrix} \right] = \\ &= \int_2^3 \frac{1}{x} - \frac{1}{x+1} dx = [\ln|x| - \ln|x+1|]_2^3 = \ln 3 - \ln 4 - \ln 2 + \ln 3 = \ln \frac{9}{8} \end{aligned}$$

b)

$$\begin{aligned} \int_{-1}^0 \frac{1}{(x-1)(x-2)} dx &= \left[\frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2) + B(x-1)}{(x-1)(x-2)} \Rightarrow \begin{matrix} A+B=0 \\ -2A-B=1 \end{matrix} \Rightarrow \begin{matrix} A=-1 \\ B=1 \end{matrix} \right] = \\ &= \int_{-1}^0 \frac{1}{x-2} - \frac{1}{x-1} dx = [\ln|x-2| - \ln|x-1|]_{-1}^0 = \ln 2 - \ln 1 - \ln 3 + \ln 2 = \ln \frac{4}{3} \end{aligned}$$

4313.

$$\begin{aligned} \int_0^1 \frac{x+4}{(x-2)(x+1)} dx &= \left[\frac{A}{x-2} + \frac{B}{x+1} = \frac{A(x+1) + B(x-2)}{(x-2)(x+1)} \Rightarrow \begin{matrix} A+B=1 \\ A-2B=4 \end{matrix} \Rightarrow \begin{matrix} A=2 \\ B=-1 \end{matrix} \right] = \\ &= \int_0^1 \frac{2}{x-2} - \frac{1}{x+1} dx = [2 \ln|x-2| - \ln|x+1|]_0^1 = 2 \ln 1 - \ln 2 - 2 \ln 2 + \ln 1 = \\ &= -3 \ln 2 \approx -2.1 \end{aligned}$$

4314. a)

$$\int \frac{2}{x+2} dx = 2 \ln|x+2| + C$$

b)

$$\begin{aligned} \int \frac{2}{x^2+2x} dx &= \int \frac{2}{x(x+2)} dx = \left[\frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2) + Bx}{x(x+2)} \Rightarrow \begin{matrix} A=1 \\ B=-1 \end{matrix} \right] = \\ &= \int \frac{1}{x} - \frac{1}{x+2} dx = \ln|x| - \ln|x+2| + C \end{aligned}$$

c)

$$\int \frac{x}{x+1} dx = \int \frac{x+1}{x+1} - \frac{1}{x+1} dx = x - \ln|x+1| + C$$

d)

$$\int \frac{2x}{x-2} dx = \int \frac{2x-4}{x-2} + \frac{4}{x-2} dx = \int 2 + \frac{4}{x-2} dx = 2x + 4 \ln|x-2| + C$$

4315.

$$\int_5^{e+4} \frac{x}{x-4} dx = \int_5^{e+4} \frac{x-4}{x-4} + \frac{4}{x-4} dx = [x + 4 \ln|x-4|]_5^{e+4} = e + 4 + 4 - 5 = e + 3$$

4316. a)

$$\begin{aligned} \int \frac{2x+1}{(x-1)(x+2)} dx &= \left[\frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2) + B(x-1)}{(x-1)(x+2)} \Rightarrow \begin{matrix} A=1 \\ B=1 \end{matrix} \right] = \\ &= \int \frac{1}{x-1} + \frac{1}{x+2} dx = \ln|x-1| + \ln|x+2| + C \end{aligned}$$

b)

$$\begin{aligned} \int \frac{2x}{x^2+4x+4} dx &= \int \frac{2x}{(x+2)^2} dx = \left[\frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{A(x+2) + B}{(x+2)^2} \Rightarrow \begin{matrix} A=2 \\ B=-4 \end{matrix} \right] = \\ &= \int \frac{2}{x+2} - \frac{4}{(x+2)^2} dx = 2 \ln|x+2| + \frac{4}{x+2} + C \end{aligned}$$

c)

$$\begin{aligned} \int \frac{2x+1}{x^2+x-6} dx &= \int \frac{2x+1}{(x+3)(x-2)} dx = \\ &= \left[\frac{A}{x+3} + \frac{B}{x-2} = \frac{A(x-2) + B(x+3)}{(x+3)(x-2)} \Rightarrow \begin{matrix} A+B=2 \\ -2A+3B=1 \end{matrix} \Rightarrow \begin{matrix} A=1 \\ B=1 \end{matrix} \right] = \\ &= \int \frac{1}{x+3} + \frac{1}{x-2} dx = \ln|x+3| + \ln|x-2| + C \end{aligned}$$

d)

$$\begin{aligned} \int \frac{4x}{(1-2x)^2} dx &= \left[\frac{A}{1-2x} + \frac{B}{(1-2x)^2} = \frac{A(1-2x) + B}{(1-2x)^2} \Rightarrow \begin{matrix} A=-2 \\ B=2 \end{matrix} \right] = \\ &= 2 \int \frac{1}{(1-2x)^2} - \frac{1}{1-2x} dx = \frac{1}{1-2x} + \ln|1-2x| + C \end{aligned}$$

4317.

$$\begin{aligned} \int_2^5 \frac{x+3}{(x-1)^2} dx &= \left[\frac{A}{(x-1)^2} + \frac{B}{x-1} = \frac{A + B(x-1)}{(x-1)^2} \Rightarrow \begin{matrix} A=4 \\ B=1 \end{matrix} \right] = \\ &= \int_2^5 \frac{4}{(x-1)^2} + \frac{1}{x-1} dx = \left[\frac{4}{1-x} + \ln|x-1| \right]_2^5 = \\ &\quad \frac{4}{-4} + \ln 4 - \frac{4}{-1} - \ln 1 = 3 + \ln 4 \end{aligned}$$

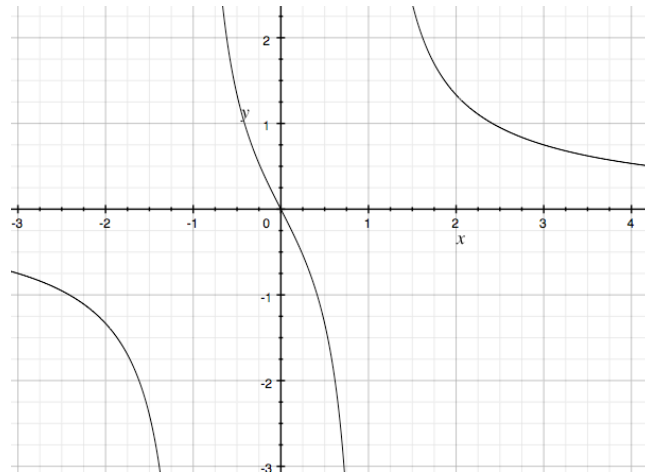
4318.

$$\begin{aligned}
\int_0^1 \frac{x+4}{(x+1)(x+2)} dx &= \left\{ \frac{A}{x+1} + \frac{B}{x+2} = \frac{A(x+2) + B(x+1)}{(x+1)(x+2)} \Rightarrow \begin{cases} A+B=1 \\ 2A+B=4 \end{cases} \Rightarrow \begin{cases} A=3 \\ B=-2 \end{cases} \right\} = \\
&= \int_0^1 \frac{3}{x+1} - \frac{2}{x+2} dx = [3 \ln|x+1| - 2 \ln|x+2|]_0^1 = \\
&= [3 \ln 2 - 2 \ln 3 - 3 \ln 1 + 2 \ln 2] = 5 \ln 2 - 2 \ln 3 = \ln \frac{32}{9}
\end{aligned}$$

4319.

$$\begin{aligned}
\int_1^e \frac{x+4}{x(x+1)} dx &= \left\{ \frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1) + Bx}{x(x+1)} \Rightarrow \begin{cases} A=4 \\ B=-3 \end{cases} \right\} = \\
&= \int_1^e \frac{4}{x} - \frac{3}{x+1} dx = [4 \ln|x| - 3 \ln|x+1|]_1^e = \\
&= [4 \ln e - 3 \ln(e+1) - 4 \ln 1 + 3 \ln 2] = 4 - 3 \ln(e+1) + 3 \ln 2 = \\
&= 4 + 3 \ln \frac{2}{e+1}
\end{aligned}$$

4320.



$$\int_2^4 \frac{2x}{x^2 - 1} dx = [\ln|x^2 - 1|]_2^4 = \ln 15 - \ln 3 = \ln 5 \text{ a. e.}$$

$$4321. \text{ a) } \int x \sin x dx = \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = \sin x \quad g = -\cos x \end{array} \right\} = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$$

$$\text{b) } \int x \cos 2x dx = \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = \cos 2x \quad g = \frac{1}{2} \sin 2x \end{array} \right\} = \frac{1}{2} x \sin 2x - \frac{1}{2} \int \sin 2x dx = \\ = \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C$$

$$\text{c) } \int x e^{0.5x} dx = \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = e^{0.5x} \quad g = 2e^{0.5x} \end{array} \right\} = 2x e^{0.5x} - 2 \int e^{0.5x} dx = 2x e^{0.5x} - 4e^{0.5x} + C$$

$$\text{d) } \int (x + 1) e^{2x} dx = \left\{ \begin{array}{l} f = x + 1 \quad f' = 1 \\ g' = e^{2x} \quad g = \frac{1}{2} e^{2x} \end{array} \right\} = (x + 1) \frac{1}{2} e^{2x} - \frac{1}{2} \int e^{2x} dx = \\ = (x + 1) \frac{1}{2} e^{2x} - \frac{1}{4} e^{2x} + C = x \frac{1}{2} e^{2x} + \frac{1}{4} e^{2x} + C$$

$$4322. \text{ a) } \int_0^1 x e^x dx = \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = e^x \quad g = e^x \end{array} \right\} = [x e^x]_0^1 - \int_0^1 e^x dx = e - [e^x]_0^1 = e - e + 1 = 1$$

$$\text{b) } \int_0^1 x \sin 2x dx = \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = \sin 2x \quad g = -\frac{1}{2} \cos 2x \end{array} \right\} = \left[x \frac{1}{2} \cos 2x \right]_{\frac{\pi}{4}}^0 + \frac{1}{2} \int_1^{\frac{\pi}{4}} \cos 2x = \frac{1}{4}$$

$$4323. \text{ a) } \int_0^{\pi} x \cos 2x dx = \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = \cos 2x \quad g = \frac{1}{2} \sin 2x \end{array} \right\} = \frac{1}{2} [x \sin 2x]_0^{\pi} - \frac{1}{2} \int_0^{\pi} \sin 2x dx = \\ = \frac{1}{4} [\cos 2x]_0^{\pi} = 0$$

$$\text{b) } \int_0^{\pi/2} x \sin x dx = \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = \sin x \quad g = -\cos x \end{array} \right\} = [x \cos x]_{\pi/2}^0 - \int_0^{\pi/2} \sin x dx = [\cos x]_{\pi/2}^0 = 1$$

4324.

$$\int_0^3 2xe^{3x} dx = \left\{ \begin{array}{l} f = 2x \quad f' = 2 \\ g' = e^{3x} \quad g = \frac{1}{3}e^{3x} \end{array} \right\} = \frac{1}{3}[2xe^{3x}]_0^3 - 2 \int_0^3 \frac{e^{3x}}{3} dx = 2e^9 - \frac{2}{9}[e^{3x}]_0^3 =$$

$$= 2e^9 - \frac{2}{9}(e^9 - 1) = \frac{16}{9}e^9 + \frac{2}{9} = \frac{2}{9}(8e^9 + 1) \approx 14\,406$$

4325.

$$\int \ln x dx = \int 1 \cdot \ln x dx = \left\{ \begin{array}{l} f = \ln x \quad f' = \frac{1}{x} \\ g' = 1 \quad g = x \end{array} \right\} = x \ln x - \int x \frac{1}{x} dx = x \ln x - x + C$$

$$4326. \text{ a) } \int x \ln x dx = \left\{ \begin{array}{l} f = \ln x \quad f' = \frac{1}{x} \\ g' = x \quad g = \frac{1}{2}x^2 \end{array} \right\} = \frac{1}{2}x^2 \ln x - \frac{1}{2} \int \frac{1}{x} x^2 dx = \frac{1}{2}x^2 \ln x - \frac{1}{2} \int x dx =$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C = \frac{x^2}{4}(2 \ln|x| - 1) + C$$

$$\text{b) } \int \ln 2x dx = \int (\ln 2 + \ln x) dx = x \ln 2 + x \ln x - x + C = x \ln |2x| - x + C$$

$$\text{c) } \int \ln x^2 dx = 2 \int \ln x dx = 2(x \ln|x| - x) + C$$

4327.

$$\int_1^2 x \ln x dx = \{4326 \text{ a)}\} = \left[\frac{x^2}{4}(2 \ln|x| - 1) \right]_1^2 = \frac{2^2}{4}(2 \ln|2| - 1) - \left(\frac{1^2}{4}(2 \ln|1| - 1) \right) =$$

$$= \frac{4}{4}(2 \ln 2 - 1) + \frac{1}{4} = 2 \ln 2 - \frac{3}{4}$$

$$4328. \text{ a) } \int_1^e x^2 \ln x dx = \left\{ \begin{array}{l} f = \ln x \quad f' = \frac{1}{x} \\ g' = x^2 \quad g = \frac{1}{3}x^3 \end{array} \right\} = \left[\frac{\ln x}{3} x^3 \right]_1^e - \frac{1}{3} \int_1^e x^2 dx =$$

$$= \left(\frac{\ln e}{3} e^3 - \frac{\ln 1}{3} 1^3 \right) - \frac{1}{9} [x^3]_1^e = \frac{e^3}{3} - \frac{1}{9}(e^3 - 1) = \frac{2}{9}e^3 + \frac{1}{9}$$

$$\text{b) } \int_1^e x^2 \ln x^2 dx = \int_1^e x^2 2 \ln x dx = 2 \cdot \{\text{svaret i 4328 a)}\} = \frac{2}{9}(2e^3 + 1)$$

$$\text{c) } \int_1^e x^2 \ln \frac{1}{x} dx = - \int_1^e x^2 \ln x dx = -1 \cdot \{\text{svaret i 4328 a)}\} = -\frac{1}{9}(2e^3 + 1)$$

4329. a)

$$\int x^2 \sin x dx = \left\{ \begin{array}{l} f = x^2 \quad f' = 2x \\ g' = \sin x \quad g = -\cos x \end{array} \right\} = -x^2 \cos x + 2 \int x \cos x dx =$$

$$= \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = \cos x \quad g = \sin x \end{array} \right\} = -x^2 \cos x + 2 \left(x \sin x - \int \sin x dx \right) + C =$$

$$= -x^2 \cos x + 2(x \sin x + \cos x) + C$$

b)

$$\begin{aligned} \int x^2 e^{2x} dx &= \left\{ \begin{array}{l} f = x^2 \quad f' = 2x \\ g' = e^{2x} \quad g = \frac{1}{2} e^{2x} \end{array} \right\} = \frac{x^2}{2} e^{2x} - \frac{1}{2} \int 2x e^{2x} dx = \\ &= \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = e^{2x} \quad g = \frac{1}{2} e^{2x} \end{array} \right\} = \frac{x^2}{2} e^{2x} - \left(\frac{x}{2} e^{2x} - \frac{1}{2} \int e^{2x} dx \right) = \\ &= \frac{x^2}{2} e^{2x} - \frac{x}{2} e^{2x} + \frac{1}{4} e^{2x} + C \end{aligned}$$

c)

$$\begin{aligned} \int x^3 e^x dx &= \left\{ \begin{array}{l} f = x^3 \quad f' = 3x^2 \\ g' = e^x \quad g = e^x \end{array} \right\} = x^3 e^x - 3 \int x^2 e^x dx = \left\{ \begin{array}{l} f = x^2 \quad f' = 2x \\ g' = e^x \quad g = e^x \end{array} \right\} = \\ &= x^3 e^x - 3 \left(x^2 e^x - 2 \int x e^x dx \right) = x^3 e^x - 3x^2 e^x + 6 \int x e^x dx = \\ &= \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = e^x \quad g = e^x \end{array} \right\} = x^3 e^x - 3x^2 e^x + 6 \left(x e^x - \int e^x dx \right) = \\ &= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C \end{aligned}$$

4330.

$$\begin{aligned} \int e^x \sin x dx &= \left\{ \begin{array}{l} f = e^x \quad f' = e^x \\ g' = \sin x \quad g = -\cos x \end{array} \right\} = -e^x \cos x + \int e^x \cos x dx = \\ &= \left\{ \begin{array}{l} f = e^x \quad f' = e^x \\ g' = \cos x \quad g = \sin x \end{array} \right\} = -e^x \cos x + e^x \sin x - \int e^x \sin x dx \Rightarrow \\ &\int e^x \sin x dx = e^x (\sin x - \cos x) - \int e^x \sin x dx \Rightarrow \\ &\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) + C \end{aligned}$$

4405.

$$\begin{aligned} y = x\sqrt{x} = x^{\frac{3}{2}} \Rightarrow y' &= \frac{3}{2}\sqrt{x} \Rightarrow \int_0^5 \sqrt{1 + \frac{9}{4}x} dx = \frac{4}{9} \cdot \frac{2}{3} \left[\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}} \right]_0^5 = \\ &= \frac{8}{27} \left(\left(1 + \frac{45}{4}\right)^{\frac{3}{2}} - 1 \right) = \frac{8}{27} \left(\left(\frac{7}{2}\right)^3 - 1 \right) = \frac{7^3 - 8}{27} = \frac{343 - 8}{27} = \frac{335}{27} \text{ l.e.} \end{aligned}$$

4406.

$$\begin{aligned} y(x) = \frac{x^3}{6} + \frac{1}{2x} \Rightarrow y'(x) &= \frac{x^2}{2} - \frac{1}{2x^2} = \frac{1}{2} \cdot \frac{x^4 - 1}{x^2} \Rightarrow y'^2(x) = \frac{1}{4x^4} (x^8 - 2x^4 + 1) = \\ &= \frac{1}{4} \left(x^4 - 2 + \frac{1}{x^4} \right) \Rightarrow \int_1^2 \sqrt{1 + \frac{1}{4} \left(x^4 - 2 + \frac{1}{x^4} \right)} dx = \int_1^2 \sqrt{\frac{1}{4} \left(x^4 + 2 + \frac{1}{x^4} \right)} dx = \\ &= \frac{1}{2} \int_1^2 \sqrt{\left(x^2 + \frac{1}{x^2} \right)^2} dx = \frac{1}{2} \int_1^2 \left(x^2 + \frac{1}{x^2} \right) dx = \frac{1}{2} \left[\frac{x^3}{3} - \frac{1}{x} \right]_1^2 = \frac{1}{2} \left(\frac{8}{3} - \frac{1}{2} - \left(\frac{1}{3} - 1 \right) \right) = \end{aligned}$$

$$= \frac{1}{2} \left(\frac{7}{3} + \frac{1}{2} \right) = \frac{17}{12} \text{ l.e.}$$

Test 4

1.

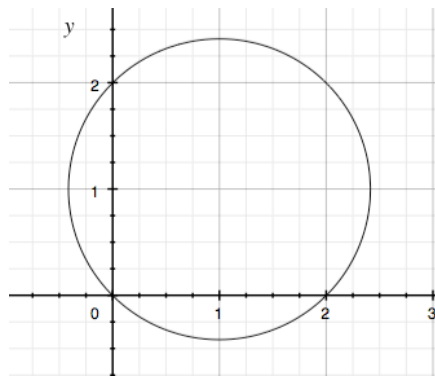
$$\int_0^{\infty} \frac{2}{e^x} dx = \lim_{w \rightarrow \infty} \int_0^w 2e^{-x} dx = 2 \lim_{w \rightarrow \infty} [e^{-x}]_0^w = 2$$

2.

$$y = (x^2 + \sin x)^3 \Rightarrow y'(x) = 3(x^2 + \sin x)^2(2x + \cos x)$$

3.

$$(x-1)^2 + (y-1)^2 = 2 \Rightarrow 2(1-x) + 2(y-1) \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1-x}{y-1}, y'(0,0) = -1$$



4.

$$\int_0^{\infty} 2e^{-0.2x} dx = \lim_{w \rightarrow \infty} \int_0^w 2e^{-0.2x} dx = 2 \lim_{w \rightarrow \infty} [5e^{-0.2x}]_0^w = 10 \text{ a. e.}$$

5.

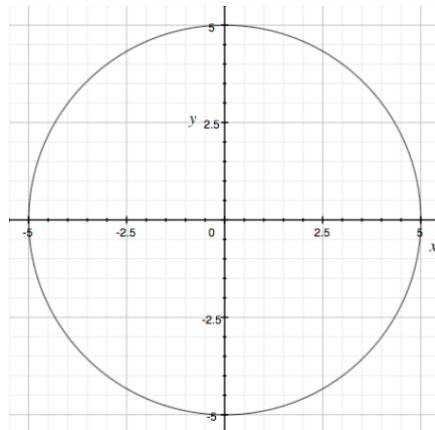
$$y^2 = 16x \Rightarrow 2y \frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y}$$

a) $y'(4) = \frac{8}{y} = 1$

b) $y = 4 + x$

c) $y = -4 - x$ (symmetriskäl)

6.



$$x^2 + y^2 = 25, \frac{d}{dx} \Rightarrow 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}, (4, \pm 3) \Rightarrow y'(4) = \pm \frac{4}{3}$$

7.

$$\frac{x}{x+4} = \frac{x+4-4}{x+4} = \frac{x+4}{x+4} + \frac{-4}{x+4} = 1 - \frac{4}{x+4}$$

8. a)

$$\int \frac{2x-3}{x} dx = \int \frac{2x}{x} - \frac{3}{x} dx = \int 2 - \frac{3}{x} dx = 2x - 3 \ln|x| + C$$

b)

$$\begin{aligned} \int \frac{5x+2}{(x+1)(x-2)} dx &= \int \frac{A}{x+1} + \frac{B}{x-2} dx = \int \frac{A(x-2) + B(x+1)}{(x+1)(x-2)} dx = \\ &= \left\{ \begin{array}{l} A+B=5 \\ -2A+B=2 \end{array} \Rightarrow \begin{array}{l} 3A=3 \\ B=4 \end{array} \right\} = \int \frac{1}{x+1} + \frac{4}{x-2} dx \end{aligned}$$

9. a)

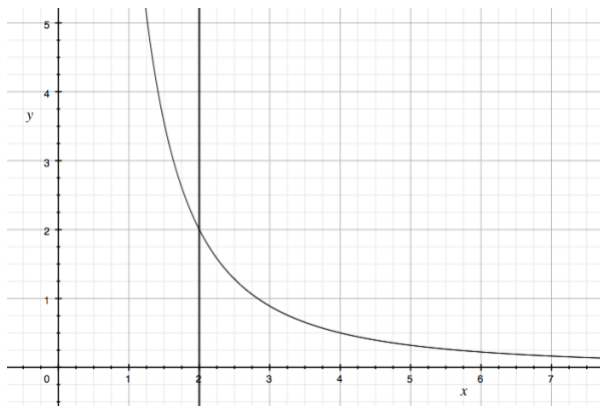
$$\int_0^1 2xe^x dx = \left\{ \begin{array}{l} f=x \quad f'=1 \\ g'=e^x \quad g=e^x \end{array} \right\} = 2[xe^x]_0^1 - 2 \int_0^1 e^x dx = 2e - 2[e^x]_0^1 = 2e - 2(e-1) = 2$$

b)

$$\begin{aligned} \int_0^{\pi/4} 3x \sin 2x dx &= \left\{ \begin{array}{l} f=x \quad f'=1 \\ g'=\sin 2x \quad g=-\frac{1}{2} \cos 2x \end{array} \right\} = \frac{3}{2} [x \cos 2x]_{\pi/4}^0 + \frac{3}{2} \int_0^{\pi/4} \cos 2x dx = \\ &= \frac{3}{2} (0 - 0) + \frac{3}{4} [\sin 2x]_0^{\pi/4} = \frac{3}{4} \end{aligned}$$

10.

$$\int_2^{\infty} \frac{8}{x^2} dx = 8 \left[\frac{1}{x} \right]_2^{\infty} = 4$$



11. a)

$$\int_0^1 -\frac{x}{2} e^{-x^2} dx = \frac{1}{4} [e^{-x^2}]_0^1 = \frac{e^{-1} - 1}{4}$$

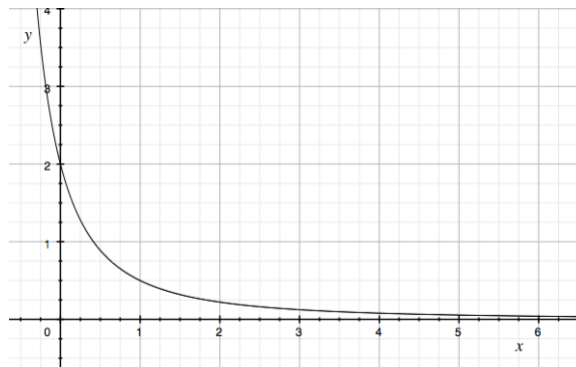
b)

$$\int_0^{\infty} -\frac{x}{2} e^{-x^2} dx = \frac{1}{4} [e^{-x^2}]_0^{\infty} = -\frac{1}{4}$$

12.

$$\int_1^e x^3 \ln x dx = \left\{ \begin{array}{l} f(x) = \ln x \quad f'(x) = \frac{1}{x} \\ g'(x) = x^3 \quad g(x) = \frac{x^4}{4} \end{array} \right\} = \left[\frac{x^4}{4} \ln x \right]_1^e - \frac{1}{4} \int_1^e x^4 \frac{1}{x} dx =$$
$$= \frac{e^4}{4} - \frac{1}{4} \int_1^e x^3 dx = \frac{e^4}{4} - \frac{1}{16} [x^4]_1^e = \frac{e^4}{4} - \frac{1}{16} (e^4 - 1) = \frac{3e^4 + 1}{16}$$

13.



$$\int_0^{\infty} \frac{4\pi}{(x+1)^4} dx = \frac{4\pi}{3} \left[\frac{1}{(x+1)^3} \right]_{\infty}^0 = \frac{4\pi}{3} \text{ v. e.}$$

14.

$$V = \frac{4\pi}{3} r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2 \Rightarrow dr = \frac{dV}{4\pi r^2} = \frac{2.4}{4\pi 1.6^2} \approx 0.075 \text{ dm/min}$$

Kapitel 4 Blandade uppgifter

3.

$$\lim_{n \rightarrow \infty} \int_1^n \frac{3dx}{x^2\sqrt{x}} = 3 \lim_{n \rightarrow \infty} \int_1^n x^{-\frac{5}{2}} dx = 3 \cdot \frac{2}{3} \lim_{n \rightarrow \infty} \left[x^{-\frac{3}{2}} \right]_1^n = 2$$

4.

$$\frac{d}{dx}(2(x+2)^2 + y^2) = \frac{d}{dx}(4) \Rightarrow 4(x+2) + 2y \frac{dy}{dx} = 0 \Rightarrow$$

$$\frac{dy}{dx} = -\frac{2(x+2)}{y} \Rightarrow y'(-2) = 0$$

5.

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(2) \Rightarrow 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} \Rightarrow y'(1) = -1 \Rightarrow y = 2 - x$$

6.

$$\frac{2x}{x+3} = \frac{2x+6-6}{x+3} = \frac{2x+6}{x+3} - \frac{6}{x+3} = 2 - \frac{6}{x+3}$$

7.

$$\begin{aligned} \frac{dr}{dt} = 4 \text{ och } A = \pi r^2 &\Rightarrow \frac{dA}{dr} = 2\pi r \Rightarrow \frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dr} \frac{dr}{dt} = 2\pi r \frac{dr}{dt} = \\ &= 2\pi \cdot 20 \cdot 4 = 160\pi \approx 500 \text{ cm}^2/\text{s} = 5 \text{ dm}^3/\text{s} \end{aligned}$$

8. Konens volym är:

$$V = \frac{hb}{3} = \frac{h\pi r^2}{3} = \{\text{men } h = 3r\} = \frac{\pi h^3}{27} \Rightarrow \frac{dV}{dh} = \frac{\pi h^2}{9}$$

$$\frac{dh}{dt} = \frac{dh}{dt} \frac{dV}{dV} = \frac{dh}{dV} \frac{dV}{dt} = \left(\frac{dV}{dh} \right)^{-1} \frac{dV}{dt} = \frac{9}{\pi h^2} \frac{dV}{dt} = \frac{9}{\pi 3^2} 1.5 \approx 4.8 \text{ cm/min}$$

9. a)

$$y = (e^{x^2} + x^2)^5 \Rightarrow y' = 5(e^{x^2} + x^2)^4 (2xe^{x^2} + 2x) = 10x(e^{x^2} + x^2)^4 (e^{x^2} + 1)$$

b)

$$y = \frac{5}{(\cos x - x^3)^2} \Rightarrow y' = \frac{5(-2)(-\sin x - 3x^2)}{(\cos x - x^3)^3} = \frac{10(\sin x + 3x^2)}{(\cos x - x^3)^3}$$

10. a)

$$x^2 + y + y^2 = 1 \Rightarrow 2x + \frac{dy}{dx} + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{1+2y}$$

b)

$$x^2 \cdot y^2 - 4 = 0, \frac{d}{dx} \Rightarrow 2x \cdot y^2 + x^2 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

11.

$$V = \int_{-1}^{\infty} \pi e^{-2(x+1)} dx = \pi \left[\frac{1}{2} e^{-2(x+1)} \right]_{\infty}^{-1} = \frac{\pi}{2} \text{ v.e.}$$

12. a)

$$\begin{aligned} \frac{5x}{(x-1)(x+4)} &= \frac{A}{x-1} + \frac{B}{x+4} = \frac{A(x+4) + B(x-1)}{(x-1)(x+4)} = \\ &= \begin{cases} A+B=5 \\ 4A-B=0 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=4 \end{cases} = \frac{1}{x-1} + \frac{4}{x+4} \end{aligned}$$

b)

$$\begin{aligned} \frac{5}{(x-1)(x+4)} &= \frac{A}{x-1} + \frac{B}{x+4} = \frac{A(x+4) + B(x-1)}{(x-1)(x+4)} = \\ &= \begin{cases} A+B=0 \\ 4A-B=5 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \end{cases} = \frac{1}{x-1} - \frac{1}{x+4} \end{aligned}$$

13. a)

$$\begin{aligned} \int \frac{x}{x+4} dx &= \int \frac{x+4-4}{x+4} dx = \int \frac{x+4}{x+4} - \frac{4}{x+4} dx = \\ &= \int 1 - \frac{4}{x+4} dx = x - 4 \ln|x+4| + C \end{aligned}$$

b)

$$\begin{aligned} \int \frac{x+2}{x+4} dx &= \int \frac{x+4-2}{x+4} dx = \int \frac{x+4}{x+4} - \frac{2}{x+4} dx = \\ &= \int 1 - \frac{2}{x+4} dx = x - 2 \ln|x+4| + C \end{aligned}$$

14. a)

$$\begin{aligned} \int_0^{0.5} 2xe^{2x} dx &= \left\{ \begin{array}{l} f = x \quad f' = 1 \\ g' = e^{2x} \quad g = \frac{1}{2} e^{2x} \end{array} \right\} = 2 \left[x \frac{1}{2} e^{2x} \right]_0^{0.5} - 2 \int_0^{0.5} \frac{1}{2} e^{2x} dx = \\ &= \frac{e}{2} - \frac{1}{2} [e^{2x}]_0^{0.5} = \frac{e}{2} - \frac{e}{2} + \frac{1}{2} = \frac{1}{2} \end{aligned}$$

b)

$$\begin{aligned} \int_0^4 \frac{2x}{\sqrt{x^2+9}} dx &= \left[2\sqrt{x^2+9} \right]_0^4 = 2(\sqrt{4^2+9} - \sqrt{0^2+9}) = \\ &= 2(5-3) = 4 \end{aligned}$$

15.

$$V = \int_{-\infty}^{\infty} \pi e^{-2|x|} dx = 2\pi \lim_{w \rightarrow \infty} \int_0^w e^{-2x} dx = 2\pi \lim_{w \rightarrow \infty} \left[\frac{1}{2} e^{-2x} \right]_0^w = \pi \text{ v. e.}$$

16.

$$y = \frac{1}{2} \ln x - \frac{x^2}{4} \Rightarrow y' = \frac{1}{2x} - \frac{x}{2} \Rightarrow (y')^2 = \frac{1}{4x^2} - \frac{1}{2} + \frac{x^2}{4}$$

$$\int_1^e \sqrt{1 + (y')^2} dx = \int_1^e \sqrt{\frac{1}{4x^2} + \frac{1}{2} + \frac{x^2}{4}} dx = \int_1^e \sqrt{\left(\frac{1}{2x} + \frac{x}{2}\right)^2} dx =$$

$$= \frac{1}{2} \int_1^e \left(\frac{1}{x} + x\right) dx = \frac{1}{2} \left[\ln x + \frac{x^2}{2} \right]_1^e = \frac{1 + e^2}{4}$$

17.

$$42 \frac{\text{cm}^2}{\text{min}} = \frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt} = \frac{dA}{dr} \frac{dr}{dt} = \frac{d}{dr} (4\pi r^2) \frac{dr}{dt} = 8\pi r \frac{dr}{dt} = 42 \text{ cm}^2/\text{min}$$

$$\frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} = \frac{dV}{dr} \frac{dr}{dt} = \frac{d}{dr} \left(\frac{4\pi r^3}{3} \right) \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Men då:

$$\frac{dr}{dt} = \frac{42}{8\pi r} \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2 \frac{42}{8\pi r} = 21r = 252 \text{ cm}^3/\text{min}$$

18.

$$y = \frac{x^3}{12} + \frac{1}{x} \Rightarrow y' = \frac{x^2}{4} - \frac{1}{x^2} \Rightarrow (y')^2 = \frac{x^4}{16} - \frac{1}{2} + \frac{1}{x^4}$$

$$S = \int_1^4 \sqrt{1 + \frac{x^4}{16} - \frac{1}{2} + \frac{1}{x^4}} dx = \int_1^4 \sqrt{\frac{x^4}{16} + \frac{1}{2} + \frac{1}{x^4}} dx =$$

$$= \int_1^4 \sqrt{\left(\frac{x^2}{4} + \frac{1}{x^2}\right)^2} dx = \int_1^4 \left(\frac{x^2}{4} + \frac{1}{x^2}\right) dx = \left[\frac{x^3}{12} - \frac{1}{x} \right]_1^4 = \frac{4^3}{12} - \frac{1}{4} - \left(\frac{1}{12} - \frac{1}{1} \right) =$$

$$= \frac{64}{12} - \frac{3}{12} - \frac{1}{12} + \frac{12}{12} = \frac{72}{12} = 6 \text{ l. e.}$$

Omfångsrika uppgifter

(sid 190 ff)

1.

$$\text{A: } \int_0^1 e^{-x} dx = [e^{-x}]_0^1 = 1 - e^{-1} \approx 0.63$$

$$\text{B: } \int_0^1 \frac{e^x}{1+e^x} dx = [\ln(1+e^x)]_0^1 = \ln(1+e) - \ln(1+1) = \ln \frac{1+e}{2} \approx 0.62$$

$$\text{C: } \int_{-1}^0 e^x \cos(e^x) dx = [\sin(e^x)]_{-1}^0 = \sin(1) - \sin(e^{-1}) \approx 0.48$$

$$2. x^2 + y^2 = 25, \frac{d}{dx} \Rightarrow 2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} = -\frac{4}{3} \text{ i } (4,3)$$

För att hitta var tangenten skär x -axeln kan m hittas:

$$y = -\frac{4}{3}x + m \Rightarrow 3 = -\frac{4}{3}4 + m \Rightarrow m = 3 + \frac{16}{3} \Rightarrow y = -\frac{4}{3}x + \frac{25}{3}$$

$$y = 0 \Rightarrow x = \frac{25}{4} = 6.25 \Rightarrow A = A_{\Delta} - A_x = \frac{6.25 \cdot 3}{2} - \pi 5^2 \frac{1}{2\pi} \arctan\left(\frac{3}{4}\right) \approx 1.33 \text{ a. e.}$$

Eller, kanske enklare, är att inse att de båda triangelarna är kongruenta 3, 4, 5-triangelar. Detta gör att den stora triangelns 3-sida sammanfaller med den lilla triangelns 4 sida. Detta ger samma svar som ovan. Och man slipper den implicita deriveringen!

18.

$$\frac{dy}{y(N-y)} = kdt \Rightarrow dy \left(\frac{A}{y} + \frac{B}{N-y} \right) = kdt \Rightarrow dy \frac{A(N-y) + By}{y(N-y)} = kdt = \left\{ \begin{array}{l} A = \frac{1}{N} \\ B = \frac{1}{N} \end{array} \right\} \Rightarrow$$

$$\frac{1}{N} dy \left(\frac{1}{y} + \frac{1}{N-y} \right) = kdt \Rightarrow \{\text{integrera båda sidor}\} \Rightarrow \frac{1}{N} (\ln y - \ln(N-y)) = kt + C$$

$$\ln \frac{y}{N-y} = N(kt + C) \Rightarrow \frac{y}{N-y} = e^{N(kt+C)} \Rightarrow y = Ne^{N(kt+C)} - ye^{N(kt+C)} \Rightarrow$$

$$y(t) = \frac{Ne^{N(kt+C)}}{1 + e^{N(kt+C)}} = \frac{Ne^{Nkt} e^{NC}}{1 + e^{Nkt} e^{NC}} = \frac{Ne^{Nkt}}{C_1 + e^{Nkt}}$$

$$19. \text{ a) } x^3 + y^3 = 3xy, \frac{d}{dx} (x^3 + y^3 = 3xy) \Rightarrow 3x^2 + 3y^2 \frac{dy}{dx} = 3y + 3x \frac{dy}{dx}$$

$$\frac{dy}{dx} (x - y^2) = x^2 - y \Rightarrow \frac{dy}{dx} = \frac{x^2 - y}{x - y^2}$$

$$\frac{dy}{dx} = 0 \text{ då } x^2 = y \Rightarrow x^3 + x^6 = 3x^3 \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \text{ och } \begin{cases} x = \sqrt[3]{2} \\ y = \sqrt[3]{4} \end{cases}$$

$$\text{b) } \frac{dy}{dx} = \infty \text{ då } x = y^2 \text{ dvs } y^6 + y^3 = 3y^3 \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \text{ och } \begin{cases} x = \sqrt[3]{4} \\ y = \sqrt[3]{2} \end{cases}$$

$$\text{c) } x^3 + y^3 = 3xy \Rightarrow x^3 + t^3 x^3 = 3xtx \Rightarrow x = \frac{3t}{1+t^3}, y = \frac{3t^2}{1+t^3}, t \neq -1$$

20. Produkten av avstånden till $(-a, 0)$ och $(a, 0)$ fås som:

$$\sqrt{(x - (-a))^2 + y^2} \cdot \sqrt{(x - a)^2 + y^2} = a^2 \Rightarrow$$

$$((x + a)^2 + y^2)((x - a)^2 + y^2) = a^4 \Rightarrow$$

$$(x^2 + 2xa + a^2 + y^2)(x^2 - 2xa + a^2 + y^2) = a^4 \Rightarrow$$

$$\begin{aligned}
 & x^4 - x^3 2a + x^2 a^2 + x^2 y^2 + \\
 & 2ax^3 - 4x^2 a^2 + 2xa^3 + 2xay^2 + \\
 & a^2 x^2 - 2xa^3 + a^4 + a^2 y^2 + \\
 & y^2 x^2 - 2xay^2 + a^2 y^2 + y^4 = a^4
 \end{aligned}$$

$$x^4 + 2x^2 y^2 + y^4 = 2x^2 a^2 - 2a^2 y^2 \Rightarrow$$

$$(x^2 + y^2)^2 = 2a^2(x^2 - y^2) \text{ VSV}$$

b)

$$\frac{d}{dx}(x^2 + y^2)^2 = \frac{d}{dx} 2a^2(x^2 - y^2)$$

$$2(x^2 + y^2) \left(2x + 2y \frac{dy}{dx} \right) = 2a^2 \left(2x - 2y \frac{dy}{dx} \right)$$

$$x(x^2 + y^2) + y(x^2 + y^2) \frac{dy}{dx} = a^2 x - a^2 y \frac{dy}{dx}$$

$$y(x^2 + y^2) \frac{dy}{dx} + a^2 y \frac{dy}{dx} = a^2 x - x(x^2 + y^2)$$

$$\frac{dy}{dx} (a^2 y + y(x^2 + y^2)) = a^2 x - x(x^2 + y^2)$$

$$\frac{dy}{dx} = 0 \text{ då } a^2 x = x(x^2 + y^2) \Rightarrow x^2 + y^2 = a^2$$

Uttrycket är en cirkel med mittpunkt i origo och radie a , dvs 4.

$$x^2 = 4^2 - 2^2 \Rightarrow \begin{cases} x = \pm 2\sqrt{3} \\ y = \pm 2 \end{cases}$$

21. Personen befinner sig på avståndet $\sqrt{20}$ km från fyren. Kägla roterar ett varv på 20 s. Kägla hastighet torde bli:

$$v = \frac{2\pi\sqrt{20}}{20} \approx 1.4 \text{ km/s}$$

(Facit ger 3.1 km/s, oklart varför.)

22. Om papprets kortsida sätts till 1 l.e. blir långsidan $\sqrt{2}$ l.e.. Kallas den skuggade triangelns vertikala katet $1 - y$ fås:

$$(1 - y)^2 + x^2 = y^2 \Leftrightarrow 1 - 2y + y^2 + x^2 = y^2 \Leftrightarrow y = \frac{1}{2}(x^2 + 1)$$

Triangelns area kan uttryckas:

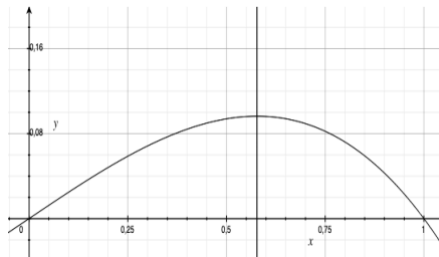
$$A(x) = \frac{1}{2}x(1 - y) = \frac{1}{2}x\left(1 - \frac{1}{2}(x^2 + 1)\right) = \frac{1}{2}x\left(\frac{1}{2} - \frac{x^2}{2}\right) = \frac{1}{4}(x - x^3)$$

Med hjälp av derivatan kan den största arean sökas:

$$\frac{dA(x)}{dx} = \frac{1}{4}(1 - 3x^2) = 0 \text{ då } x = \frac{1}{\sqrt{3}} \approx 0.58$$

Dvs

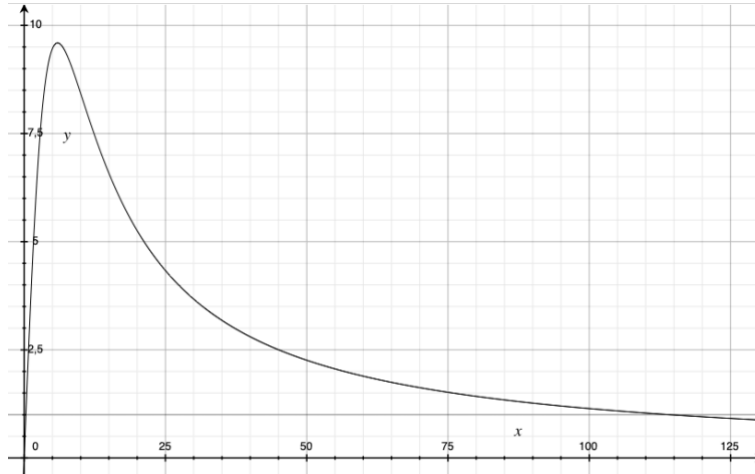
$$A_{max} = \frac{1}{4}x(1 - x^2) = \frac{1}{4} \frac{1}{\sqrt{3}} \left(1 - \frac{1}{3}\right) = \frac{1}{6\sqrt{3}} \approx 0.096 \text{ a. e.}$$



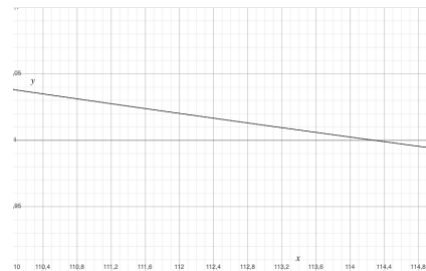
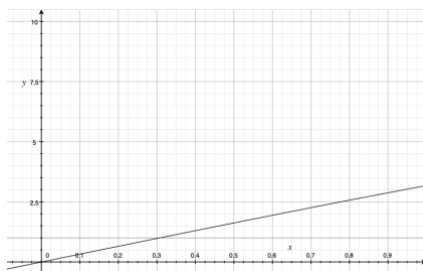
23. Funktionen $v(d)$ visar vinkelns beroende av avståndet d .

a)

$$v(d) = \arctan \frac{7}{d} - \arctan \frac{5}{d}$$



b) Detaljstudier gällande intervallen $0 < d < 1$ m och $110 < d < 115$ m nedan:



Tillsammans ger dessa intervallerna $0.3 < d < 114$ m.

c) Cirka 5-6 meter.

d)

$$v'(d) = -\frac{7}{d^2} \frac{1}{1 + \left(\frac{7}{d}\right)^2} + \frac{5}{d^2} \frac{1}{1 + \left(\frac{5}{d}\right)^2} = 0$$

$$\frac{7}{d^2} \frac{1}{1 + \left(\frac{7}{d}\right)^2} = \frac{5}{d^2} \frac{1}{1 + \left(\frac{5}{d}\right)^2} \Rightarrow 7 \left(1 + \left(\frac{5}{d}\right)^2\right) = 5 \left(1 + \left(\frac{7}{d}\right)^2\right) \Rightarrow$$

$$7 \left(1 + \frac{25}{d^2}\right) = 5 \left(1 + \frac{49}{d^2}\right) \Rightarrow 7 + \frac{175}{d^2} = 5 + \frac{245}{d^2} \Rightarrow 2 = \frac{70}{d^2} \Rightarrow d = \sqrt{35} \approx 5.9 \text{ m}$$

24.

$$\frac{\int_{-2}^0 x^3 - x^2 - 4x + 3 - (-x^2 + 3) dx}{\int_0^2 -x^2 + 3 - (x^3 - x^2 - 4x + 3) dx} = \frac{\int_{-2}^0 x^3 - 4x dx}{-\int_0^2 x^3 - 4x dx} = \frac{\left[\frac{x^4}{4} - 2x^2\right]_{-2}^0}{\left[\frac{x^4}{4} - 2x^2\right]_2^0} =$$

$$= \frac{\left[\frac{x^4}{4} - 2x^2\right]_{-2}^0}{\left[\frac{x^4}{4} - 2x^2\right]_2^0} = \frac{4}{4} = 1$$

27. a)

$$\frac{d}{dx} \cosh x = \frac{d}{dx} \left(\frac{e^x - e^{-x}}{2} \right) = \frac{e^x + e^{-x}}{2} = \sinh x$$

$$\frac{d}{dx} \sinh x = \frac{d}{dx} \left(\frac{e^x + e^{-x}}{2} \right) = \frac{e^x - e^{-x}}{2} = \cosh x$$

$$\frac{d}{dx} \tanh x = \frac{d}{dx} \left(\frac{e^x + e^{-x}}{e^x - e^{-x}} \right) = \frac{e^x - e^{-x}}{e^x - e^{-x}} - \frac{e^x + e^{-x}}{(e^x - e^{-x})^2} (e^x + e^{-x}) =$$

$$= \frac{e^{2x} - 2 + e^{-2x}}{(e^x - e^{-x})^2} - \frac{e^{2x} + 2 + e^{-2x}}{(e^x - e^{-x})^2} = -\frac{2^2}{(e^x - e^{-x})^2} = -\frac{1}{\sinh^2 x}$$

En intressant integral, Gaussklockan:

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx$$

Låt oss se om vi kan hitta I^2 :

$$\begin{aligned} I^2 &= \left(\int_{-\infty}^{\infty} e^{-x^2} dx \right)^2 = \int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-y^2} dy e^{-x^2} dx = \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(y^2+x^2)} dy dx = \left\{ \begin{array}{l} dydx = \rho d\varphi d\rho \\ y^2 + x^2 = \rho^2 \\ 0 \leq \varphi \leq 2\pi \\ 0 \leq \rho \leq \infty \end{array} \right\} = \int_0^{\infty} \int_0^{2\pi} e^{-\rho^2} \rho d\rho d\varphi = \\ &= 2\pi \frac{1}{2} [e^{-\rho^2}]_{\infty}^0 = \pi \Rightarrow I = \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \end{aligned}$$

Talkodning:

$$s_w(n) = s(n) + \sum_{i=1}^{10} a_i \gamma_1^i s(n-i) - \sum_{i=1}^{10} a_i \gamma_2^i s_w(n-i), \quad n = 0 \dots L-1$$

$$s_w(n) = s(n) + a_1 \gamma_1 s(n-1) + \dots + a_{10} \gamma_1^{10} s(n-10)$$

$$- (a_1 \gamma_2 s_w(n-1) + \dots + a_{10} \gamma_2^{10} s_w(n-10))$$

där, $n = 0 \dots 159$

eller:

$$s_w(n) = s(n) + a_1 0.9_1 s(n-1) + \dots + a_{10} 0.9^{10} s(n-10)$$

$$- (a_1 0.6 s_w(n-1) + \dots + a_{10} 0.6^{10} s_w(n-10))$$

$$D = \sum_{n=1}^{40} (x(n) - gx(n-L))^2 = \sum x(n)^2 - 2g \sum x(n)x(n-L) + \sum g^2 x^2(n-L)$$

$$\frac{\partial D}{\partial g} = -2 \sum x(n)x(n-L) + 2g \sum x^2(n-L) = 0$$

$$g = \frac{\sum x(n)x(n-L)}{\sum x^2(n-L)}$$

$$D = \sum x^2(n) - 2g \sum x(n)x(n-L) + \sum g^2 x^2(n-L) =$$

$$= \sum x^2(n) - 2 \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} + \left[\frac{\sum x(n)x(n-L)}{\sum x^2(n-L)} \right]^2 \sum x^2(n-L) =$$

$$= \sum x^2(n) - 2 \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} + \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} =$$

$$= \sum x^2(n) - \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)}$$

Testa $41 \leq L \leq 169$