Matematik 5 svar

Kapitel 4

4101. a) $y = (2x + 3)^4 \Rightarrow y'(x) = 4(2x + 3)^3 2 = 8(2x + 3)^3$

b) $y = (1 - 4x)^5 \Rightarrow y'(x) = -5 \cdot 4(1 - 4x)^4 = -20(1 - 4x)^4$

c) $y = (1 + 2x^3)^4 \Rightarrow y'(x) = 4 \cdot 2 \cdot 3x^2(1 + 2x^3)^3 = 24x^2(1 + 2x^3)^3$

d) $y = (x^2 + 2x)^3 \Rightarrow y'(x) = 3(x^2 + 2x)^2(2x + 2) = 6(x + 1)(x^2 + 2x)^2$

e) $y = \left(1 + \frac{1}{2x^3}\right)^4 = \left(1 + \frac{1}{2}x^{-3}\right)^4 \Rightarrow y'(x) = 4\left(1 + \frac{1}{2x^3}\right)^3 \frac{-3}{2} x^{-4} = -\frac{6}{x^4}\left(1 + \frac{1}{2x^3}\right)^3$

f) $y = (\sin x + \cos x)^3 \Rightarrow y'(x) = 3(\cos x - \sin x)(\sin x + \cos x)^2$

4102. a) $f(x) = (3x + 5)^{20} \Rightarrow f'(x) = 20(3x + 5)^{19}3x = 60x(3x + 5)^{19}$

b) $g(x) = \sqrt{3x - 5} \Rightarrow g'(x) = \frac{3}{2\sqrt{3x - 5}}$

c) $h(x) = \frac{1}{(2x+1)^2} = (2x + 1)^{-2} \Rightarrow h'(x) = -2 \cdot 2(2x + 1)^{-3} = -\frac{4}{(2x+1)^3}$

d) $f(t) = (e^{3t} + t)^4 \Rightarrow f'(t) = 4(3e^{3t} + 1)(e^{3t} + t)^3$

e) $g(u) = \frac{2}{e^{3u}} = 2e^{-3u} \Rightarrow g'(u) = -6e^{-3u} = -\frac{6}{e^{3u}}$

f) $h(v) = \frac{1}{\sqrt{v^2+3}} = (v^2 + 3)^{-\frac{1}{2}} \Rightarrow h'(x) = -\frac{1}{2}2v(v^2 + 3)^{-\frac{3}{2}} = -v(v^2 + 3)^{-\frac{3}{2}} = \frac{-v}{(v^2+3)^{1.5}}$

4103. a) $f(x) = e^{x^2} \Rightarrow f'(x) = 2xe^{x^2}$

b) $f(v) = \sin^2 v \Rightarrow f'(x) = 2 \sin v \cos v = \sin 2v$

4104. a) $y = (\sin x + x)^2 \Rightarrow y'(x) = 2(\sin x + x)(\cos x + 1)$

b) $y = (2x + e^{2x})^4 \Rightarrow y'(x) = 4(2x + e^{2x})^3(2 + 2e^{2x})$

c) $y = e^{(x^2+1)^2} \Rightarrow y'(x) = 2(x^2 + 1)2xe^{(x^2+1)^2} = 4x(x^2 + 1)e^{(x^2+1)^2}$

d)

$$y = \ln(x + 2x^3) \Rightarrow y'(x) = \frac{1 + 6x^2}{x + 2x^3}$$

4105. A: $f(x) = \ln e^{3x} = 3x \ln e = 3x \Rightarrow f'(x) = 3$

B:

$$f(x) = \ln(\cos 3x) \Rightarrow f'(x) = -\frac{3 \sin 3x}{\cos 3x} = -3 \tan 3x$$

4106. a)

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = 5x^2$$

b)

$$\frac{dz}{dx} = \frac{dy}{dx} \cdot \frac{dz}{dy} = \frac{dy}{dx} \cdot \left(\frac{dy}{dz}\right)^{-1} = 8 \cdot \frac{1}{2} = 4$$

4107. a) $y = x^3(x^2 - 1)^3 \Rightarrow y' = 3x^2(x^2 - 1)^3 + x^3 3(x^2 - 1)^2 2x =$

$$= 3x^2(x^2 - 1)^3 + 6x^4(x^2 - 1)^2 = 3x^2(x^2 - 1)^2(3x^2 - 1)$$

b) $y = (1 + x^2)(x^2 - 1)^3 \Rightarrow y' = 2x(x^2 - 1)^3 + (1 + x^2)3 \cdot 2x(x^2 - 1)^2 =$

$$= 2x(x^2 - 1)^2[x^2 - 1 + (1 + x^2)3] = 2x(x^2 - 1)^2[x^2 - 1 + 3 + 3x^2] =$$

$$= 4x(x^2 - 1)^2(2x^2 + 1)$$

c) $= e^{x^2}(x^3 - 1)^2 \Rightarrow y' = 2xe^{x^2}(x^3 - 1)^2 + e^{x^2}2 \cdot 3x^2(x^3 - 1)=$

$$= 2xe^{x^2}(x^3 - 1)(x^3 - 1 + 3x)$$

d) $y = \sin 2x(\cos 4x + 2) \Rightarrow y' = 2\cos 2x(\cos 4x + 2) - 4\sin 2x\sin 4x =$

$$= 2\cos 2x(\cos 4x + 2) - 4\sin 2x \cdot 2\sin 2x\cos 2x = 2\cos 2x(\cos 4x + 2 - 4\sin^2 2x)$$

4108. $\frac{ds}{dt} = 0.3\,\frac{\text{cm}}{\text{h}}$, $V = s^3 \Rightarrow \frac{dV}{dS} = 3s^2$ och $\frac{dV}{dt} = \frac{dV}{dt}\frac{ds}{ds} = \frac{dV}{ds}\frac{ds}{dt} = 3s^2 0.3 \approx 5.1 \text{ dm}^3/\text{h}$

4109. $A = x^2, \frac{dA}{dt} = 2x\frac{ds}{ds} = 2 \cdot 15 \cdot 3.7 = 111\,\frac{cm^2}{s} \approx 1.1 \text{ dm}^2/\text{s}$

4110. $50 \text{ m}^2 \cdot \frac{dx}{dt} = 0.09\,\frac{\text{m}^3}{\text{min}} \Rightarrow \frac{dx}{dt} = \frac{0.09}{50}\frac{\text{m}^3}{\text{min}}\frac{1}{\text{m}^2} = 1.8\,\frac{\text{mm}}{\text{min}}$

4111. $V = A \cdot h \Rightarrow \frac{dV}{dt} = A\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{A}\frac{dV}{dt} = \frac{1}{\pi 1.2^2}0.065\,\frac{\text{m}}{\text{min}} \approx 1.4 \text{ cm/min}$

4112. $V = \frac{4\pi r^3}{3}, \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt} = \frac{1}{4\pi 25^2}1600 \approx 2 \text{ mm/s}$

4113. $\frac{h}{r} = 1.5, \frac{dh}{dt} = \frac{dh}{dt}\frac{dV}{dV} = \frac{dV}{dt}\frac{dh}{dV} = \frac{dV}{dt}\left(\frac{dV}{dh}\right)^{-1}$ men $V = \frac{bh}{3} = \frac{\pi r^2 h}{3} = \frac{\pi\left(\frac{h}{1.5}\right)^2 h}{3} = \frac{\pi h^3}{6.75}$

$$\frac{dV}{dh} = \frac{\pi 3h^2}{6.75} = \frac{\pi h^2}{2.25} \Rightarrow \frac{dh}{dt} = \frac{dV}{dt}\left(\frac{dV}{dh}\right)^{-1} = 2 \cdot \frac{2.25}{\pi 2^2} \approx 3.6 \text{ cm/min}$$

4114. $A = \pi r^2, \frac{dA}{dt} = \frac{dA}{dt}\frac{dr}{dr} = \frac{dA}{dr}\frac{dr}{dt} = \pi 2r \cdot \frac{dr}{dt} = \pi 2 \cdot (4 \cdot 1.2) \cdot 1.2 \approx 36 \text{ dm}^2/\text{s}$

4115.

$$\frac{1}{2} = \frac{h}{d} = \frac{h}{2r} \Rightarrow h = r$$

$$\frac{dh}{dt} = \frac{dh}{dt}\frac{dV}{dV} = \frac{dV}{dt}\frac{dh}{dV} = \frac{dV}{dt}\left(\frac{dV}{dh}\right)^{-1} \text{ men } V = \frac{bh}{3} = \frac{\pi r^2 h}{3} = \frac{\pi h^2 h}{3} = \frac{\pi h^3}{3}$$

$$\frac{dV}{dh} = \frac{\pi 3h^2}{3} = \pi h^2 \Rightarrow \frac{dh}{dt} = 0.4 \cdot \frac{1}{\pi 2^2} = \frac{0.1}{\pi} \approx 3.2 \text{ cm/min}$$

4116. a)
$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{dV}{dt}\left(\frac{dV}{dr}\right)^{-1} = \frac{dV}{dt}\left(\frac{d}{dr}\frac{4\pi r^3}{3}\right)^{-1} \Rightarrow$$
$$\frac{dr}{dt} = \frac{dV}{dt}(4\pi r^2)^{-1} = \frac{150}{4\pi 25^2} \approx 0.19 \text{ mm/min}$$
b)
$$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt} = \frac{dr}{dt}\left(\frac{d}{dr}4\pi r^2\right) = \frac{150}{4\pi 25^2} \cdot 8\pi 25 = 12 \text{ cm}^2/\text{min}$$

4117. Konens volym finner man som:

$$V_{\text{kon}} = \frac{bh}{3} = \frac{\pi r^2 h}{3} = \{\text{toppvinkel} = 90° \Rightarrow r = h\} = \frac{\pi h^3}{3}$$

$$\frac{dV}{dh} = \pi h^2 \text{ dvs } \frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \left(\frac{dV}{dh}\right)^{-1}\frac{dV}{dt} = \frac{1}{\pi 3^2}2.5 \approx 8.8 \text{ mm/min}$$

4118.
$$V = \frac{4\pi r^3}{3}, A = 4\pi r^2, \frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} \text{ är konstant}$$

4119. $y^2 = 2x$ {implicit derivering ger}$2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y} \Rightarrow y'(2) = -\frac{1}{2}$

4120. a) $y^2 = 9x$ {implicit derivering ger} $2y\frac{dy}{dx} = 9 \Rightarrow \frac{dy}{dx} = \frac{9}{2y} \Rightarrow y'(4) = \frac{9}{12} = \frac{3}{4}$

b) Enpunktsformeln ger $y - 6 = \frac{3}{4}(x - 4) \Rightarrow y = \frac{3}{4}x + 6 - 4\frac{3}{4} = \frac{3}{4}x + 3$

c) $y = -\frac{3}{4}x - 3$ (fel i facit)

4121. a) $x^2 + 2y^2 = 1$ {implicit derivering ger} $2x + 4y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{2y}$

b) $x^2 + 2y = y^3$ {implicit derivering ger} $2x + 2\frac{dy}{dx} = 3y^2\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{2x}{3y^2 - 2}$

4122. $x^2 + y^2 = 25$ {implicit derivering ger}$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

Lutningen i $(3,4)$ är $-3/4$.

4123. $2x^2 + 3y^2 = 12$ {implicit derivering ger} $4x + 6y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{3y}$

4124. $2x^2 + 3y^2 = 14$ {implicit derivering ger} $4x + 6y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{3y} = -\frac{1}{3}$

4125. $x^2 + y^2 = 2$ {implicit derivering ger} $2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

Enpunktsformeln ger: $y - (-1) = -\frac{-1}{-1}\big(x - (-1)\big) \Rightarrow y = -x - 2$

4127.

$$\frac{d}{dx}(x^2 \cdot y) = 0 \Rightarrow 2x \cdot y + x^2\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2xy}{x^2} = -\frac{2}{x} \cdot y = -\frac{2}{x^3}$$
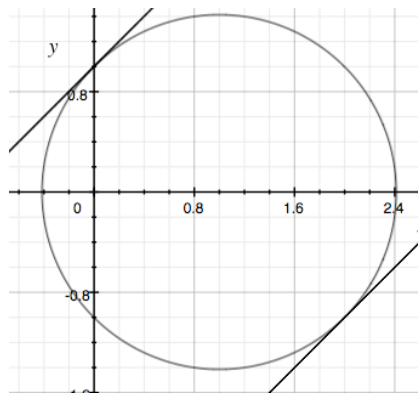
$$y = \frac{1}{x^2} \Rightarrow y' = -\frac{2}{x^3}$$

Ja, samma resultat.

4128.

$$(x-1)^2 + y^2 = 2 \quad \frac{d}{dx} \Rightarrow 2(x-1) + 2y\frac{dy}{dx} = 0 \text{ dvs då } 1-x = y$$
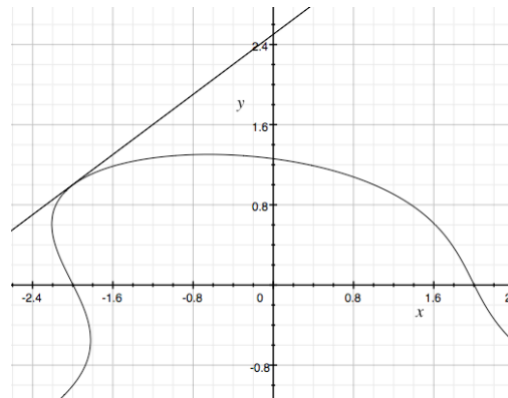$$y^2 + y^2 = 2 \Rightarrow (x_1, y_1) = (0,1) \text{ och } (x_2, y_2) = (2,-1)$$

4129.

$$x^2 + xy + 2y^3 = 4 \quad \frac{d}{dx} \Rightarrow 2x + y + x\frac{dy}{dx} + 2 \cdot 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 6y^2) = -2x - y \Rightarrow \frac{dy}{dx} = -\frac{2x + y}{x + 6y^2} = -\frac{-4 + 1}{-2 + 6} = \frac{3}{4}$$

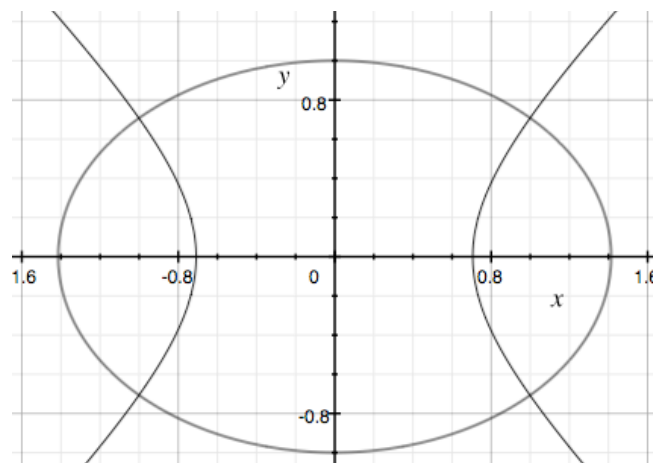$$y = kx + m \Rightarrow 1 = \frac{3}{4}(-2) + m \Rightarrow m = 1 + \frac{3}{2} = 2.5$$

$$y = 0.75x + 2.5$$

4130.

$$\begin{cases} x^2 + 2y^2 = 2 \\ 2x^2 - 2y^2 = 1 \end{cases} \Rightarrow 3x^2 = 3 \Rightarrow \begin{cases} x = \pm 1 \\ y = \pm\dfrac{1}{\sqrt{2}} \end{cases}$$

$$\frac{d}{dx}\begin{cases} x^2 + 2y^2 = 2 \\ 2x^2 - 2y^2 = 1 \end{cases} \Rightarrow \begin{cases} 2x + 4y\dfrac{dy}{dx} = 0 \\ 4x - 4y\dfrac{dy}{dx} = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{dy}{dx} = -\dfrac{x}{2y} = -\dfrac{1}{\sqrt{2}} \\ \dfrac{dy}{dx} = \dfrac{x}{y} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2} \end{cases} \text{VSV}$$
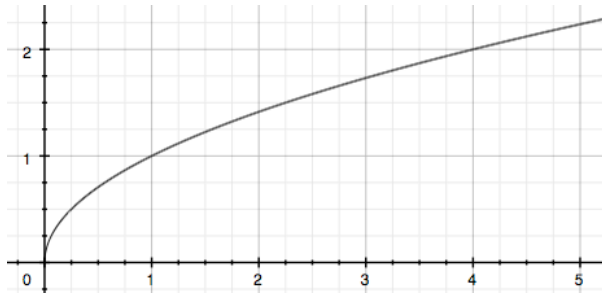
4131.

$$\frac{d}{dt}(y^2 = 300^2 + x^2) \Rightarrow 2y\frac{dy}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{x}{\sqrt{300^2 + x^2}}50 \text{ m/min}$$

a) $\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{300}{\sqrt{300^2 + 300^2}}50 \approx 35$ m/min

b) $\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{400}{\sqrt{300^2 + 400^2}}50 = 40$ m/min

4132.

$$\frac{d}{dt}(y^2 = x) \Rightarrow 2y\frac{dy}{dt} = \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{1}{2y}\frac{dx}{dt} = \frac{1.5}{4} = \frac{3}{8} \approx 0.38 \text{ m/s}$$

4133.

$$x^2 = 1500^2 + y^2, \qquad \frac{d}{dt}(x^2 = 1500^2 + y^2) \Rightarrow 2x\frac{dx}{dt} = 2y\frac{dy}{dt} \Rightarrow$$

$$\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt} = \frac{1000}{\sqrt{1500^2 + 1000^2}}200 \approx 110 \text{ m/s}$$

4134.

$$\frac{d}{dt}(y^2 = 5^2 - x^2) \Rightarrow 2y\frac{dy}{dt} = -2x\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{3}{\sqrt{5^2 - 3^2}}0.04 = -0.03 \text{ m/s}$$

4135.

$$v = 540\frac{\text{km}}{\text{h}} = 150 \text{ m/s}$$

$$a(t) = \sqrt{(vt)^2 + h^2} \Rightarrow \frac{da}{dt} = \frac{1}{2}\frac{2tv^2}{\sqrt{(vt)^2 + h^2}} = \{t = 60 \text{ s}\} \approx 130\frac{\text{m}}{\text{s}} \approx 470 \text{ km/h}$$

4201. a)

$$\int_0^\infty e^{-x}dx = \lim_{t\to\infty}\int_0^t e^{-x}dx = \lim_{t\to\infty}[e^{-x}]_t^0 = 1$$

b)

$$\int_0^\infty e^{-2x}dx = \lim_{t\to\infty}\int_0^t e^{-2x}dx = \frac{1}{2}\lim_{t\to\infty}[e^{-2x}]_t^0 = \frac{1}{2}$$

c)

$$\int_0^\infty \sin x\, dx = \lim_{t\to\infty}\int_0^t \sin x\, dx = \lim_{t\to\infty}[\cos x]_0^t \Rightarrow \text{divergent}$$

4202. a)

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_1^\infty \Rightarrow \text{divergent}$$

b)

$$\int_1^\infty \frac{1}{x}\,dx = \left[\ln x\right]_1^\infty \Rightarrow \text{divergent}$$

c)

$$\int_1^\infty \frac{1}{x^2}\,dx = \left[\frac{1}{x}\right]_\infty^1 = 1$$

4203.

$$A = \int_0^\infty e^{-0.2x}\,dx = \left[5e^{-0.2x}\right]_\infty^0 = 5 \text{ a. e.}$$

4204. a)

$$\lim_{w\to\infty} \int_2^w \frac{2}{x^5}\,dx = \lim_{w\to\infty} \left[-\frac{2}{4x^4}\right]_2^w = \frac{1}{2}\lim_{w\to\infty}\left[\frac{1}{2^4} - \frac{1}{w^4}\right] = \frac{1}{32}$$

b)

$$\lim_{w\to\infty} \int_0^w \frac{4}{(2x+1)^3}\,dx = \lim_{w\to\infty}\left[-\frac{1}{2}\cdot\frac{1}{2}\frac{4}{(2x+1)^2}\right]_0^w = \lim_{w\to\infty}\left[1 - \frac{1}{(2w+1)^2}\right] = 1$$

c)

$$\lim_{w\to\infty} \int_0^w 3e^{-2x}\,dx = \lim_{w\to\infty}\left[-\frac{3}{2}e^{-2x}\right]_0^w = \frac{3}{2}$$

d)

$$\lim_{w\to\infty} \int_4^w \frac{2}{x\sqrt{x}}\,dx = \lim_{w\to\infty}\left[-2\frac{2}{\sqrt{x}}\right]_4^w = 2$$

e)

$$\lim_{w\to\infty} \int_1^w \frac{2}{\sqrt{x}}\,dx = \lim_{w\to\infty}\left[2\cdot 2\sqrt{x}\right]_1^w \text{ divergerar}$$

f)

$$\lim_{w\to\infty} \int_{-w}^1 e^{2x}\,dx = \lim_{w\to\infty}\left[\frac{1}{2}e^{2x}\right]_{-w}^1 = \frac{e^2}{2}$$
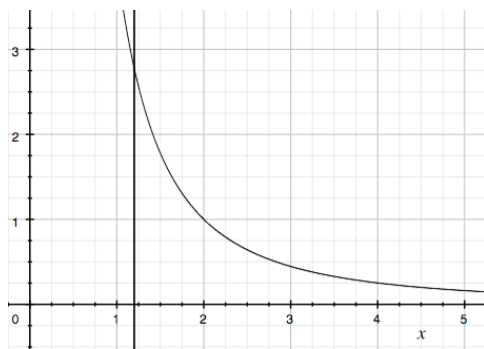
4205.

$$\lim_{a\to\infty} \int_0^a 4e^{-0.9x}\,dx = \lim_{a\to\infty}\left[-\frac{4}{0.9}e^{-0.9x}\right]_0^a = \frac{4}{0.9}$$

4206.

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_0^1 = 2$$

4207.

$$\lim_{w \to \infty} \int_a^w \frac{4}{x^2} dx = \lim_{w \to \infty} \left[-\frac{4}{x}\right]_a^w = \frac{4}{a} = 4 \Rightarrow a = 1$$

4208.

$$\lim_{w \to \infty} \int_1^w \frac{1}{x^k} dx = \lim_{w \to \infty} \int_1^w x^{-k} dx = \lim_{w \to \infty} \left[\frac{1}{-k+1} x^{-k+1}\right]_1^w = \frac{1}{1-k} \lim_{w \to \infty} \left[\frac{1}{x^{k-1}}\right]_1^w =$$

$$= \frac{1}{1-k} \lim_{w \to \infty} \left(\frac{1}{w^{k-1}} - 1\right) \text{ gränsvärdet konvergerar om } k > 1.$$

4209.

$$V = \int_1^5 \pi(2x)^2 dx = 4\pi \int_1^5 x^2 dx = \frac{4\pi}{3} [x^3]_1^5 = \frac{4\pi 124}{3} \approx 519 \text{ v. e.}$$

Volymen kallas en stympad kon.

4212.

$$V = \lim_{w \to \infty} \int_0^w \pi e^{-2x} dx = \frac{\pi}{2} \lim_{w \to \infty} [e^{-2x}]_w^0 = \frac{\pi}{2} \text{ v. e.}$$

4213. Volymen är den som i vänsterkant begränsas av $x = 2$.

$$V = \lim_{w \to \infty} \int_2^w \pi \left(\frac{1}{x\sqrt{x}}\right)^2 dx = \pi \lim_{w \to \infty} \int_2^w \frac{1}{x^3} dx = \frac{\pi}{2} \lim_{w \to \infty} \left[\frac{1}{x^2}\right]_w^2 = \frac{\pi}{8} \text{ v. e.}$$

4214. Volymen är den som i vänsterkant begränsas av $x = 2$.

$$V = \lim_{w \to \infty} \int_2^w \frac{\pi dx}{(x-1)^2} = \pi \lim_{w \to \infty} \left[\frac{1}{x-1}\right]_w^2 = \pi \text{ v. e.}$$

4215. a)
$$\lim_{w\to\infty} \int_1^w \frac{1}{x}dx = \lim_{w\to\infty} [\ln|x|]_1^w \text{ divergent} \Rightarrow \text{oänligt mycket färg!}$$

b)
$$\lim_{w\to\infty} \int_1^w \pi \frac{1}{x^2}dx = \pi \lim_{w\to\infty} \left[\frac{1}{x}\right]_w^1 = \pi \text{ v. e. färg.}$$

4216.
$$\int_{-1}^3 \pi(x+1)dx = \pi \left[\frac{x^2}{2}+x\right]_{-1}^3 = \pi\left(\frac{9}{2}+3-\frac{1}{2}+1\right) = 8\pi \text{ v. e.}$$

4217.
$$\int_{-1}^a \pi(x+1)dx = \pi\left[\frac{x^2}{2}+x\right]_{-1}^a = \pi\left(\frac{a^2}{2}+a-\frac{1}{2}+1\right) = 16\pi$$
$$\frac{a^2}{2}+a-\frac{1}{2}+1 = 16 \Rightarrow a^2+2a-31=0$$
$$a = -1 \pm \sqrt{1+31} - 1 \pm \sqrt{32} = 4\sqrt{2}-1 \approx 4.7$$

4218. a)
$$\int_0^2 10e^{-0.5t}dt = 10\cdot 2\,[e^{-0.5t}]_2^0 = 20(1-e^{-1}) \approx 12.6 \text{ g}$$

b) 20 g

4219.
$$\lim_{w\to\infty} \int_0^w 100e^{-0.05t}dt = 2000 \lim_{w\to\infty} [e^{-0.05t}]_w^0 = 2000 \text{ liter}$$

4220.
$$\lim_{w\to\infty} \int_0^w 2e^{-0.025x}dx = 80 \lim_{w\to\infty} [e^{-0.025t}]_w^0 = 80$$
Den ursprungliga temperaturen var 80°.

4221.
$$\lim_{w\to\infty} \int_0^w 0.001e^{-0.001x}dx = \lim_{w\to\infty} [e^{-0.001t}]_w^0 = 1$$
Sannolikheten att den förr eller senare går sönder = 1.

4222. a)
$$5200 \int_0^2 \frac{dt}{(1+t)^2} = 5200 \left[\frac{1}{1+t}\right]_2^0 = 5200\left(1-\frac{1}{3}\right) \approx 3500 \text{ st}$$

b) 5200 st

4223.

$$\lim_{w\to\infty}\int_0^w 0.001e^{-0.05t}\,dt = 0.02\lim_{w\to\infty}[e^{-0.05t}]_w^0 = 0.02 \text{ g}$$

Facit svarar μg, troligen skulle det varit μg i uppgiften.

4224.

$$i(t) = 4.41\cdot 10^{-5}\cdot 0.956^t = 4.41\cdot 10^{-5}\cdot e^{t\ln 0.956}$$

$$q_{tot} = 4.41\cdot 10^{-5}\lim_{w\to\infty}\int_0^\infty e^{t\ln 0.956}\,dt = \frac{4.41\cdot 10^{-5}}{\ln 0.956}\lim_{w\to\infty}[e^{t\ln 0.956}]_0^w =$$

$$= \frac{4.41\cdot 10^{-5}}{\ln 0.956}(-1) = 0.98 \text{ mC}$$

4225. a)

$$\int_{R_j}^{R} F(r)\,dr = 6.672\cdot 10^{-11}M\cdot m\int_{R_j}^{R}\frac{dr}{r^2} = 6.672\cdot 10^{-11}M\cdot m\left[\frac{1}{r}\right]_R^{R_j} =$$

$$= 6.672\cdot 10^{-11}M\cdot m\left(\frac{1}{R_j}-\frac{1}{R}\right) =$$

$$= 6.672\cdot 10^{-11}\cdot 5.9735\cdot 10^{24}\cdot 15\,000\left(\frac{1}{6.37\cdot 10^6}-\frac{1}{6.38\cdot 10^6}\right)\approx 1.47 \text{ GJ}$$

Energin som krävs att lyfta 15 ton 10 km ut från jordens yta.

b) Cirka 940 GJ för att lyfta 15 ton bort från jordens gravitationsfält.

4301. a) $\frac{x^2+3}{x^2} = 1 + \frac{3}{x^2}$ b) $\frac{10-x}{2x} = \frac{10}{2x} - \frac{x}{2x} = \frac{5}{x} - \frac{1}{2}$ c) $\frac{x(x+2)+1}{x+2} = \frac{x(x+2)}{x+2} + \frac{1}{x+2} = x + \frac{1}{x+2}$

4302. a) $\frac{x}{x+4} = \frac{x+4-4}{x+4} = \frac{x+4}{x+4} - \frac{4}{x+4} = 1 - \frac{4}{x+4}$ b) $\frac{x}{x-7} = \frac{x-7+7}{x-7} = \frac{x-7}{x-7} + \frac{7}{x-7} = 1 + \frac{7}{x-7}$

4303. a) $\frac{x}{x+1} = \frac{x+1-1}{x+1} = \frac{x+1}{x+1} - \frac{1}{x+1} = 1 - \frac{1}{x+1}$ b) $\frac{x}{x-5} = \frac{x-5+5}{x-5} = \frac{x-5}{x-5} + \frac{5}{x-5} = 1 + \frac{5}{x-5}$

c) $\frac{x}{x+8} = \frac{x+8-8}{x+8} = \frac{x+8}{x+8} - \frac{8}{x+8} = 1 - \frac{8}{x+8}$

4304. $\frac{1}{t(t+1)} = \frac{a}{t} + \frac{b}{t+1} = \frac{a(t+1)}{t(t+1)} + \frac{bt}{(t+1)t} = \frac{at+a+bt}{t(t+1)} = \left\{\begin{matrix}a=1\\b=-1\end{matrix}\right\} = \frac{1}{t} - \frac{1}{t+1}$

4305.

$$\frac{3x}{(x+2)(x-1)} = \frac{a}{x+2} + \frac{b}{x-1} = \frac{a(x-1)}{(x+2)(x-1)} + \frac{b(x+2)}{(x+2)(x-1)} =$$

$$= \frac{a(x-1)+b(x+2)}{(x+2)(x-1)} = \left\{\begin{matrix}a+b=3\\-a+2b=0\end{matrix}\right\} \Rightarrow \left\{\begin{matrix}b=1\\a=2\end{matrix}\right. = \frac{2}{x+2} + \frac{1}{x-1}$$

4306. a)

$$\frac{x+1}{x+2} = \frac{x+1+1-1}{x+2} = \frac{x+2-1}{x+2} = \frac{x+2}{x+2} - \frac{1}{x+2} = 1 - \frac{1}{x+2}$$

b)

$$\frac{x-2}{x+1} = \frac{x-2+3-3}{x+1} = \frac{x+1-3}{x+1} = \frac{x+1}{x+1} - \frac{3}{x+1} = 1 - \frac{3}{x+1}$$

c)

$$\frac{x+1}{x-3} = \frac{x+1-4+4}{x-3} = \frac{x-3}{x-3} + \frac{4}{x-3} = 1 + \frac{4}{x-3}$$

4307. a)

$$\frac{x+1}{(x-2)(x-1)} = \frac{a}{x-2} + \frac{b}{x-1} = \frac{a(x-1)}{(x-2)(x-1)} + \frac{b(x-2)}{(x-2)(x-1)} =$$

$$= \frac{a(x-1)+b(x-2)}{(x-2)(x-1)} = \left\{ \begin{matrix} a+b=1 \\ -a-2b=1 \end{matrix} \Rightarrow \begin{matrix} b=-2 \\ a=3 \end{matrix} \right\} =$$

$$= \frac{3}{x-2} - \frac{2}{x-1}$$

b)

$$\frac{4}{x(x-4)} = \frac{a}{x} + \frac{b}{x-4} = \frac{a(x-4)}{x(x-4)} + \frac{bx}{x(x-4)} = \frac{a(x-4)+bx}{x(x-4)} =$$

$$= \left\{ \begin{matrix} a=-1 \\ b=1 \end{matrix} \right\} = \frac{1}{x-4} - \frac{1}{x}$$

c)

$$\frac{2x+5}{x^2-5x} = \frac{2x+5}{x(x-5)} = \frac{a}{x} + \frac{b}{x-5} = \frac{a(x-5)+bx}{x(x-5)} = \left\{ \begin{matrix} a+b=2 \\ a=-1 \end{matrix} \Rightarrow \begin{matrix} a=-1 \\ b=3 \end{matrix} \right\} =$$

$$= \frac{3}{x-5} - \frac{1}{x}$$

4308.

$$\frac{1-x}{(x+2)^2} = \frac{a}{x+2} + \frac{b}{(x+2)^2} = \frac{a(x+2)+b}{(x+2)^2} = \left\{ \begin{matrix} a=-1 \\ b=3 \end{matrix} \right\} = \frac{3}{(x+2)^2} - \frac{1}{x+2}$$

4309. a)

$$\frac{x}{(x-1)^2} = \frac{a}{(x-1)^2} + \frac{b}{x-1} = \frac{a+b(x-1)}{(x-1)^2} = \left\{ \begin{matrix} b=1 \\ a=1 \end{matrix} \right\} = \frac{1}{(x-1)^2} + \frac{1}{x-1}$$

b)

$$\frac{x+3}{(x-1)^2} = \frac{a}{(x-1)^2} + \frac{b}{x-1} = \frac{a+b(x-1)}{(x-1)^2} = \left\{ \begin{matrix} b=1 \\ a=4 \end{matrix} \right\} = \frac{4}{(x-1)^2} + \frac{1}{x-1}$$

4310. a)

$$\int_1^3 \frac{3x+x^2}{x} dx = \int_1^3 3 + x\, dx = \left[3x + \frac{x^2}{2} \right]_1^3 = 9 + \frac{9}{2} - 3 - \frac{1}{2} = 10$$

b)

$$\int_1^2 \frac{2+x}{x} dx = \int_1^2 \frac{2}{x} + 1 = [2\ln x + x]_1^2 = 2\ln 2 + 2 - 2\ln 1 - 1 = 2\ln 2 + 1$$

4311. a)
$$\int \frac{x}{x+2}\,dx = \int \frac{x+2}{x+2} - \frac{2}{x+2}\,dx = x - 2\ln|x+2| + C$$

b)
$$\int \frac{x}{x-2}\,dx = \int \frac{x-2}{x-2} + \frac{2}{x-2}\,dx = x + 2\ln|x-2| + C$$

4312. a)
$$\int_{2}^{3} \frac{1}{x(x+1)}\,dx = \left[\frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{x(x+1)} \Rightarrow \begin{matrix} A = 1 \\ B = -1 \end{matrix}\right] =$$

$$= \int_{2}^{3} \frac{1}{x} - \frac{1}{x+1}\,dx = [\ln|x| - \ln|x+1|]_{2}^{3} = \ln 3 - \ln 4 - \ln 2 + \ln 3 = \ln\frac{9}{8}$$

b)
$$\int_{-1}^{0} \frac{1}{(x-1)(x-2)}\,dx = \left[\frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)} \Rightarrow \begin{matrix} A+B = 0 \\ -2A-B = 1 \end{matrix} \Rightarrow \begin{matrix} A = -1 \\ B = 1 \end{matrix}\right] =$$

$$= \int_{-1}^{0} \frac{1}{x-2} - \frac{1}{x-1}\,dx = [\ln|x-2| - \ln|x-1|]_{-1}^{0} = \ln 2 - \ln 1 - \ln 3 + \ln 2 = \ln\frac{4}{3}$$

4313.
$$\int_{0}^{1} \frac{x+4}{(x-2)(x+1)}\,dx = \left[\frac{A}{x-2} + \frac{B}{x+1} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)} \Rightarrow \begin{matrix} A+B = 1 \\ A-2B = 4 \end{matrix} \Rightarrow \begin{matrix} A = 2 \\ B = -1 \end{matrix}\right] =$$

$$= \int_{0}^{1} \frac{2}{x-2} - \frac{1}{x+1}\,dx = [2\ln|x-2| - \ln|x+1|]_{0}^{1} = 2\ln 1 - \ln 2 - 2\ln 2 + \ln 1 =$$

$$= -3\ln 2 \approx -2.1$$

4314. a)
$$\int \frac{2}{x+2}\,dx = 2\ln|x+2| + C$$

b)
$$\int \frac{2}{x^2 + 2x}\,dx = \int \frac{2}{x(x+2)}\,dx = \left[\frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)+Bx}{x(x+2)} \Rightarrow \begin{matrix} A = 1 \\ B = -1 \end{matrix}\right] =$$

$$= \int \frac{1}{x} - \frac{1}{x+2}\,dx = \ln|x| - \ln|x+2| + C$$

c)

$$\int \frac{x}{x+1}\,dx = \int \frac{x+1}{x+1} - \frac{1}{x+1}\,dx = x - \ln|x+1| + C$$

d)

$$\int \frac{2x}{x-2}\,dx = \int \frac{2x-4}{x-2} + \frac{4}{x-2}\,dx = \int 2 + \frac{4}{x-2}\,dx = 2x + 4\ln|x-2| + C$$

4315.

$$\int_5^{e+4} \frac{x}{x-4}\,dx = \int_5^{e+4} \frac{x-4}{x-4} + \frac{4}{x-4}\,dx = [x + 4\ln|x-4|]_5^{e+4} = e + 4 + 4 - 5 = e + 3$$

4316. a)

$$\int \frac{2x+1}{(x-1)(x+2)}\,dx = \left[\frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2) + B(x-1)}{(x-1)(x+2)} \Rightarrow \begin{matrix} A = 1 \\ B = 1 \end{matrix} \right] =$$

$$= \int \frac{1}{x-1} + \frac{1}{x+2}\,dx = \ln|x-1| + \ln|x+2| + C$$

b)

$$\int \frac{2x}{x^2 + 4x + 4}\,dx = \int \frac{2x}{(x+2)^2}\,dx = \left[\frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{A(x+2) + B}{(x+2)^2} \Rightarrow \begin{matrix} A = 2 \\ B = -4 \end{matrix} \right] =$$

$$= \int \frac{2}{x+2} - \frac{4}{(x+2)^2}\,dx = 2\ln|x+2| + \frac{4}{x+2} + C$$

c)

$$\int \frac{2x+1}{x^2 + x - 6}\,dx = \int \frac{2x+1}{(x+3)(x-2)}\,dx =$$

$$= \left[\frac{A}{x+3} + \frac{B}{x-2} = \frac{A(x-2) + B(x+3)}{(x+3)(x-2)} \Rightarrow \begin{matrix} A + B = 2 \\ -2A + 3B = 1 \end{matrix} \Rightarrow \begin{matrix} A = 1 \\ B = 1 \end{matrix} \right] =$$

$$= \int \frac{1}{x+3} + \frac{1}{x-2}\,dx = \ln|x+3| + \ln|x-2| + C$$

d)

$$\int \frac{4x}{(1-2x)^2}\,dx = \left[\frac{A}{1-2x} + \frac{B}{(1-2x)^2} = \frac{A(1-2x) + B}{(1-2x)^2} \Rightarrow \begin{matrix} A = -2 \\ B = 2 \end{matrix} \right] =$$

$$= 2\int \frac{1}{(1-2x)^2} - \frac{1}{1-2x}\,dx = \frac{1}{1-2x} + \ln|1-2x| + C$$

4317.

$$\int_2^5 \frac{x+3}{(x-1)^2}\,dx = \left[\frac{A}{(x-1)^2} + \frac{B}{x-1} = \frac{A + B(x-1)}{(x-1)^2} \Rightarrow \begin{matrix} A = 4 \\ B = 1 \end{matrix} \right] =$$

$$= \int_2^5 \frac{4}{(x-1)^2} + \frac{1}{x-1}\,dx = \left[\frac{4}{1-x} + \ln|x-1| \right]_2^5 =$$

$$\frac{4}{-4} + \ln 4 - \frac{4}{-1} - \ln 1 = 3 + \ln 4$$
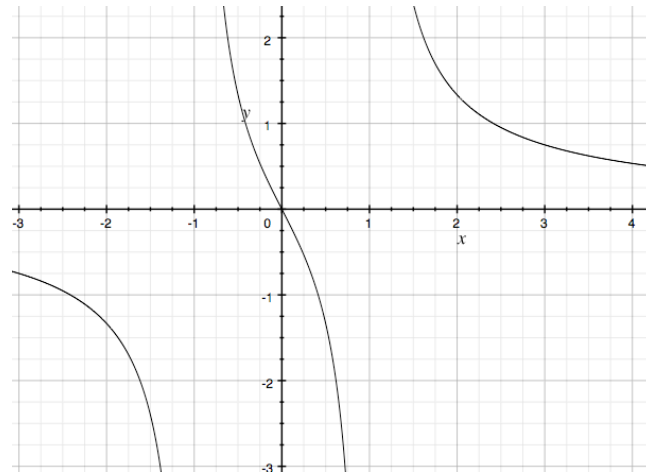
4318.

$$\int_0^1 \frac{x+4}{(x+1)(x+2)}dx = \left\{\frac{A}{x+1} + \frac{B}{x+2} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)} \Rightarrow \left\{\begin{array}{l} A+B=1 \\ 2A+B=4 \end{array}\right. \Rightarrow \left\{\begin{array}{l} A=3 \\ B=-2 \end{array}\right.\right\} =$$

$$= \int_0^1 \frac{3}{x+1} - \frac{2}{x+2}dx = [3\ln|x+1| - 2\ln|x+2|]_0^1 =$$

$$= [3\ln 2 - 2\ln 3 - 3\ln 1 + 2\ln 2] = 5\ln 2 - 2\ln 3 = \ln\frac{32}{9}$$

4319.

$$\int_1^e \frac{x+4}{x(x+1)}dx = \left\{\frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{x(x+1)} \Rightarrow \left\{\begin{array}{l} A=4 \\ B=-3 \end{array}\right.\right\} =$$

$$= \int_1^e \frac{4}{x} - \frac{3}{x+1}dx = [4\ln|x| - 3\ln|x+1|\,]_1^e =$$

$$= [4\ln e - 3\ln(e+1) - 4\ln 1 + 3\ln 2] = 4 - 3\ln(e+1) + 3\ln 2 =$$

$$= 4 + 3\ln\frac{2}{e+1}$$

4320.

$$\int_{2}^{4} \frac{2x}{x^2-1}dx = [\ln|x^2-1|]_{2}^{4} = \ln 15 - \ln 3 = \ln 5 \ \text{a.e.}$$

4321. a) $\int x \sin x\, dx = \left\{ \begin{matrix} f = x & f' = 1 \\ g' = \sin x & g = -\cos x \end{matrix} \right\} = -x \cos x + \int \cos x\, dx = -x \cos x + \sin x + C$

b) $\int x \cos 2x\, dx = \left\{ \begin{matrix} f = x & f' = 1 \\ g' = \cos 2x & g = \frac{1}{2}\sin 2x \end{matrix} \right\} = \frac{1}{2}x \sin 2x - \frac{1}{2}\int \sin 2x\, dx =$

$$= \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C$$

c) $\int x e^{0.5x}dx = \left\{ \begin{matrix} f = x & f' = 1 \\ g' = e^{0.5x} & g = 2e^{0.5x} \end{matrix} \right\} = 2xe^{0.5x} - 2\int e^{0.5x}dx = 2xe^{0.5x} - 4e^{0.5x} + C$

d) $\int (x+1)e^{2x}dx = \left\{ \begin{matrix} f = x+1 & f' = 1 \\ g' = e^{2x} & g = \frac{1}{2}e^{2x} \end{matrix} \right\} = (x+1)\frac{1}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx =$

$$= (x+1)\frac{1}{2}e^{2x} - \frac{1}{4}e^{2x} + C = x\frac{1}{2}e^{2x} + \frac{1}{4}e^{2x} + C$$

4325.

$$\int \ln x\, dx = \int 1 \cdot \ln x\, dx = \left\{ \begin{matrix} f = \ln x & f' = \frac{1}{x} \\ g' = 1 & g = x \end{matrix} \right\} = x \ln x - \int x \frac{1}{x}dx = x \ln x - x + C$$

4326. a) $\int x \ln x\, dx = \left\{ \begin{matrix} f = \ln x & f' = \frac{1}{x} \\ g' = x & g = \frac{1}{2}x^2 \end{matrix} \right\} = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int \frac{1}{x}x^2\, dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\, dx =$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C = \frac{x^2}{4}(2\ln|x| - 1) + C$$

b) $\int \ln 2x\, dx = \int (\ln 2 + \ln x)dx = x \ln 2 + x \ln x - x + C = x \ln |2x| - x + C$

c) $\int \ln x^2\, dx = 2\int \ln x\, dx = 2(x \ln|x| - x) + C$

4327.

$$\int_1^2 x \ln x \, dx = \{4326 \ a)\} = \left[\frac{x^2}{4}(2\ln|x| - 1)\right]_1^2 = \frac{2^2}{4}(2\ln|2| - 1) - \left(\frac{1^2}{4}(2\ln|1| - 1)\right) =$$

$$= \frac{4}{4}(2\ln 2 - 1) + \frac{1}{4} = 2\ln 2 - \frac{3}{4}$$

4328. a) $\int_1^e x^2 \ln x \, dx = \begin{Bmatrix} f = \ln x & f' = \frac{1}{x} \\ g' = x^2 & g = \frac{1}{3}x^3 \end{Bmatrix} = \left[\frac{\ln x}{3}x^3\right]_1^e - \frac{1}{3}\int_1^e x^2 dx =$

$$= \left(\frac{\ln e}{3}e^3 - \frac{\ln 1}{3}1^3\right) - \frac{1}{9}[x^3]_1^e = \frac{e^3}{3} - \frac{1}{9}(e^3 - 1) = \frac{2}{9}e^3 + \frac{1}{9}$$

b) $\int_1^e x^2 \ln x^2 \, dx = \int_1^e x^2 2\ln x \, dx = 2 \cdot \{\text{svaret i } 4328 \ a)\} = \frac{2}{9}(2e^3 + 1)$

c) $\int_1^e x^2 \ln\frac{1}{x} dx = -\int_1^e x^2 \ln x \, dx = -1 \cdot \{\text{svaret i } 4328 \ a)\} = -\frac{1}{9}(2e^3 + 1)$

4329. a)

$$\int x^2 \sin x \, dx = \begin{Bmatrix} f = x^2 & f' = 2x \\ g' = \sin x & g = -\cos x \end{Bmatrix} = -x^2 \cos x + 2\int x \cos x \, dx =$$

$$= \begin{Bmatrix} f = x & f' = 1 \\ g' = \cos x & g = \sin x \end{Bmatrix} = -x^2 \cos x + 2\left(x\sin x - \int \sin x \, dx\right) + C =$$

$$= -x^2 \cos x + 2(x\sin x + \cos x) + C$$

b)

$$\int x^2 e^{2x} dx = \begin{Bmatrix} f = x^2 & f' = 2x \\ g' = e^{2x} & g = \frac{1}{2}e^{2x} \end{Bmatrix} = \frac{x^2}{2}e^{2x} - \frac{1}{2}\int 2xe^{2x} dx =$$

$$= \begin{Bmatrix} f = x & f' = 1 \\ g' = e^{2x} & g = \frac{1}{2}e^{2x} \end{Bmatrix} = \frac{x^2}{2}e^{2x} - \left(\frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x} dx\right) =$$

$$= \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C$$

c)

$$\int x^3 e^x dx = \begin{Bmatrix} f = x^3 & f' = 3x^2 \\ g' = e^x & g = e^x \end{Bmatrix} = x^3 e^x - 3\int x^2 e^x dx = \begin{Bmatrix} f = x^2 & f' = 2x \\ g' = e^x & g = e^x \end{Bmatrix} =$$

$$= x^3 e^x - 3\left(x^2 e^x - 2\int xe^x dx\right) = x^3 e^x - 3x^2 e^x + 6\int xe^x dx =$$

$$= \begin{Bmatrix} f = x & f' = 1 \\ g' = e^x & g = e^x \end{Bmatrix} = x^3 e^x - 3x^2 e^x + 6\left(xe^x - \int e^x dx\right) =$$

$$= x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + C$$

4330.

$$\int e^x \sin x \, dx = \begin{Bmatrix} f = e^x & f' = e^x \\ g' = \sin x & g = -\cos x \end{Bmatrix} = -e^x \cos x + \int e^x \cos x \, dx =$$

$$= \begin{Bmatrix} f = e^x & f' = e^x \\ g' = \cos x & g = \sin x \end{Bmatrix} = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx \Rightarrow$$

$$\int e^x \sin x \, dx = e^x(\sin x - \cos x) - \int e^x \sin x \, dx \Rightarrow$$

$$\int e^x \sin x \, dx = \frac{1}{2}e^x(\sin x - \cos x) + C$$

4405.

$$y = x\sqrt{x} = x^{\frac{3}{2}} \Rightarrow y' = \frac{3}{2}\sqrt{x} \Rightarrow \int_0^5 \sqrt{1 + \frac{9}{4}x}\,dx = \frac{4}{9}\cdot\frac{2}{3}\left[\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}}\right]_0^5 =$$

$$= \frac{8}{27}\left(\left(1 + \frac{45}{4}\right)^{\frac{3}{2}} - 1\right) = \frac{8}{27}\left(\left(\frac{7}{2}\right)^3 - 1\right) = \frac{7^3 - 8}{27} = \frac{343 - 8}{27} = \frac{335}{27} \text{ l.e.}$$

4406.

$$y(x) = \frac{x^3}{6} + \frac{1}{2x} \Rightarrow y'(x) = \frac{x^2}{2} - \frac{1}{2x^2} = \frac{1}{2}\cdot\frac{x^4 - 1}{x^2} \Rightarrow y'^2(x) = \frac{1}{4x^4}(x^8 - 2x^4 + 1) =$$

$$= \frac{1}{4}\left(x^4 - 2 + \frac{1}{x^4}\right) \Rightarrow \int_1^2 \sqrt{1 + \frac{1}{4}\left(x^4 - 2 + \frac{1}{x^4}\right)}\,dx = \int_1^2 \sqrt{\frac{1}{4}\left(x^4 + 2 + \frac{1}{x^4}\right)}\,dx =$$

$$= \frac{1}{2}\int_1^2 \sqrt{\left(x^2 + \frac{1}{x^2}\right)^2}\,dx = \frac{1}{2}\int_1^2 x^2 + \frac{1}{x^2}\,dx = \frac{1}{2}\left[\frac{x^3}{3} - \frac{1}{x}\right]_1^2 = \frac{1}{2}\left(\frac{8}{3} - \frac{1}{2} - \left(\frac{1}{3} - 1\right)\right) =$$

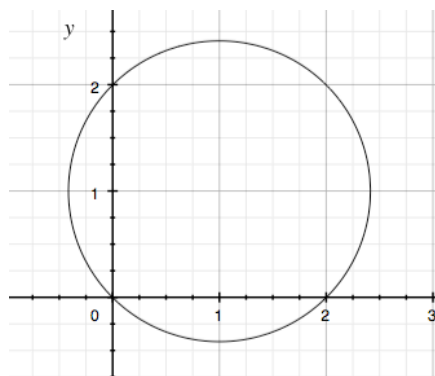$$= \frac{1}{2}\left(\frac{7}{3} + \frac{1}{2}\right) = \frac{17}{12} \text{ l.e.}$$

Test 4

1.

$$\int_0^\infty \frac{2}{e^x}\,dx = \lim_{w\to\infty} \int_0^w 2e^{-x}\,dx = 2\lim_{w\to\infty}[e^{-x}]_w^0 = 2$$

2.

$$y = (x^2 + \sin x)^3 \Rightarrow y'(x) = 3(x^2 + \sin x)^2(2x + \cos x)$$

3.

$$(x-1)^2 + (y-1)^2 = 2 \Rightarrow 2(1-x) + 2(y-1)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1-x}{y-1}, y'(0,0) = -1$$

4.

$$\int_0^\infty 2e^{-0.2}\,dx = \lim_{w\to\infty}\int_0^w 2e^{-0.2}\,dx = 2\lim_{w\to\infty}[5e^{-0.2}]_w^0 = 10 \text{ a. e.}$$
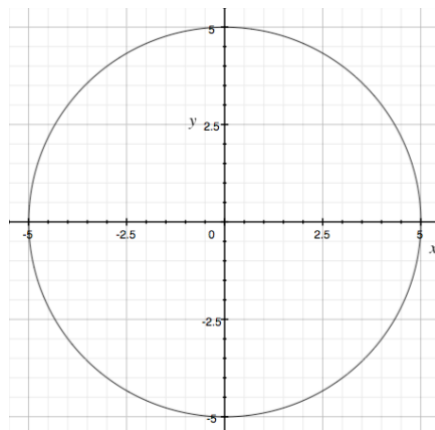
5.

$$y^2 = 16x \Rightarrow 2y\frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y}$$

a) $y'(4) = \frac{8}{y} = 1$ b) $y = 4 + x$ c) $y = -4 - x$ (symmetriskäl)

6.

$$x^2 + y^2 = 25, \frac{d}{dx} \Rightarrow 2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}, (4, \pm 3) \Rightarrow y'(4) = \pm\frac{4}{3}$$

7.

$$\frac{x}{x+4} = \frac{x+4-4}{x+4} = \frac{x+4}{x+4} + \frac{-4}{x+4} = 1 - \frac{4}{x+4}$$

8. a)

$$\int \frac{2x-3}{x} dx = \int \frac{2x}{x} - \frac{3}{x} dx \int 2 - \frac{3}{x} dx = 2x - 3\ln|x| + C$$

b)

$$\int \frac{5x+2}{(x+1)(x-2)} dx = \int \frac{A}{x+1} + \frac{B}{x-2} dx = \int \frac{A(x-2) + B(x+1)}{(x+1)(x-2)} dx =$$

$$= \left\{ \begin{matrix} A+B=5 \\ -2A+B=2 \end{matrix} \Rightarrow \begin{matrix} 3A=3 \\ B=4 \end{matrix} \right\} = \int \frac{1}{x+1} + \frac{4}{x-2} dx$$
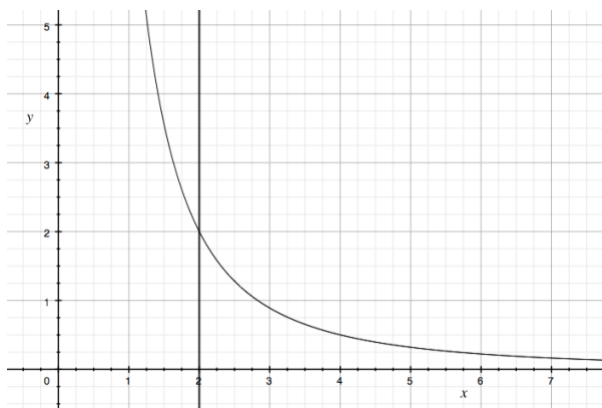
9. a)

$$\int_0^1 2xe^x dx = \left\{ \begin{matrix} f = x & f' = 1 \\ g' = e^x & g = e^x \end{matrix} \right\} = 2[xe^x]_0^1 - 2\int_0^1 e^x dx = 2e - 2[e^x]_0^1 = 2e - 2(e-1) = 2$$

b)

$$\int_0^{\pi/4} 3x \sin 2x \, dx = \left\{ \begin{matrix} f = x & f' = 1 \\ g' = \sin 2x & g = -\frac{1}{2}\cos 2x \end{matrix} \right\} = \frac{3}{2}[x\cos 2x]_{\pi/4}^0 + \frac{3}{2}\int_0^{\pi/4} \cos 2x \, dx =$$

$$= \frac{3}{2}(0-0) + \frac{3}{4}[\sin 2x]_0^{\pi/4} = \frac{3}{4}$$

10.

$$\int_2^\infty \frac{8}{x^2} dx = 8\left[\frac{1}{x}\right]_\infty^2 = 4$$

11. a)
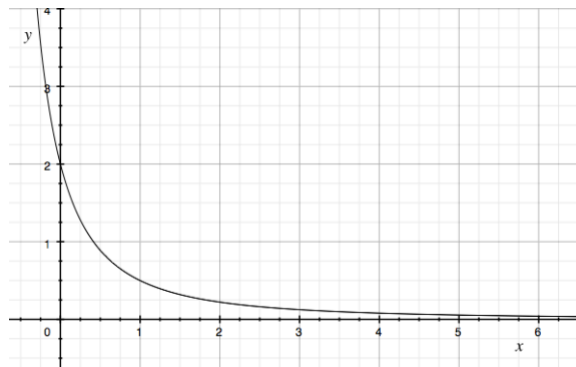$$\int_0^1 -\frac{x}{2}e^{-x^2}\,dx = \frac{1}{4}\left[e^{-x^2}\right]_0^1 = \frac{e^{-1}-1}{4}$$

b)
$$\int_0^{\infty} -\frac{x}{2}e^{-x^2}\,dx = \frac{1}{4}\left[e^{-x^2}\right]_0^{\infty} = -\frac{1}{4}$$

12.
$$\int_1^e x^3 \ln x\,dx = \begin{cases} f(x) = \ln x & f'(x) = \dfrac{1}{x} \\ g'(x) = x^3 & g(x) = \dfrac{x^4}{4} \end{cases} = \left[\frac{x^4}{4}\ln x\right]_1^e - \frac{1}{4}\int_1^e x^4\frac{1}{x}\,dx =$$

$$= \frac{e^4}{4} - \frac{1}{4}\int_0^e x^3\,dx = \frac{e^4}{4} - \frac{1}{16}\left[x^4\right]_1^e = \frac{e^4}{4} - \frac{1}{16}(e^4-1) = \frac{3e^4+1}{16}$$

13.

$$\int_0^{\infty} \frac{4\pi}{(x+1)^4}\,dx = \frac{4\pi}{3}\left[\frac{1}{(x+1)^3}\right]_{\infty}^0 = \frac{4\pi}{3} \text{ v.e.}$$

14.
$$V = \frac{4\pi}{3}r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2 \Rightarrow dr = \frac{dV}{4\pi r^2} = \frac{2.4}{4\pi 1.6^2} \approx 0.075 \text{ dm/min}$$

Kapitel 4 Blandade uppgifter

3.

$$\lim_{n\to\infty} \int_1^n \frac{3dx}{x^2\sqrt{x}} = 3 \lim_{n\to\infty} \int_1^n x^{-\frac{5}{2}}dx = 3 \cdot \frac{2}{3} \lim_{n\to\infty} \left[x^{-\frac{3}{2}} \right]_n^1 = 2$$

4.

$$\frac{d}{dx}(2(x+2)^2 + y^2) = \frac{d}{dx}(4) \Rightarrow 4(x+2) + 2y\frac{dy}{dx} = 0 \Rightarrow$$

$$\frac{dy}{dx} = -\frac{2(x+2)}{y} \Rightarrow y'(-2) = 0$$

5.

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(2) \Rightarrow 2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} \Rightarrow y'(1) = -1 \Rightarrow y = 2 - x$$

6.

$$\frac{2x}{x+3} = \frac{2x+6-6}{x+3} = \frac{2x+6}{x+3} - \frac{6}{x+3} = 2 - \frac{6}{x+3}$$

7.

$$\frac{dr}{dt} = 4 \text{ och } A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r \Rightarrow \frac{dA}{dt} = \frac{dA}{dt}\frac{dr}{dr} = \frac{dA}{dr}\frac{dr}{dt} = 2\pi r \frac{dr}{dt} =$$

$$= 2\pi \cdot 20 \cdot 4 = 160\pi \approx 500 \text{ cm}^2/\text{s} = 5 \text{ dm}^3/\text{s}$$

8. Konens volym är:

$$V = \frac{hb}{3} = \frac{h\pi r^2}{3} = \{\text{men } h = 3r\} = \frac{\pi h^3}{27} \Rightarrow \frac{dV}{dh} = \frac{\pi h^2}{9}$$

$$\frac{dh}{dt} = \frac{dh}{dt}\frac{dV}{dV} = \frac{dh}{dV}\frac{dV}{dt} = \left(\frac{dV}{dh}\right)^{-1}\frac{dV}{dt} = \frac{9}{\pi h^2}\frac{dV}{dt} = \frac{9}{\pi 3^2}1.5 \approx 4.8 \text{ cm/min}$$

9. a)

$$y = \left(e^{x^2} + x^2\right)^5 \Rightarrow y' = 5\left(e^{x^2} + x^2\right)^4\left(2xe^{x^2} + 2x\right) = 10x\left(e^{x^2} + x^2\right)^4\left(e^{x^2} + 1\right)$$

b)

$$y = \frac{5}{(\cos x - x^3)^2} \Rightarrow y' = \frac{5(-2)(-\sin x - 3x^2)}{(\cos x - x^3)^3} = \frac{10(\sin x + 3x^2)}{(\cos x - x^3)^3}$$

10. a)

$$x^2 + y + y^2 = 1 \Rightarrow 2x + \frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{1+2y}$$

b)

$$x^2 \cdot y^2 - 4 = 0, \frac{d}{dx} \Rightarrow 2x \cdot y^2 + x^2 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

11.

$$V = \int_{-1}^{\infty} \pi e^{-2(x+1)} dx = \pi \left[\frac{1}{2} e^{-2(x+1)}\right]_{\infty}^{-1} = \frac{\pi}{2} \text{ v.e.}$$

12. a)

$$\frac{5x}{(x-1)(x+4)} = \frac{A}{x-1} + \frac{B}{x+4} = \frac{A(x+4) + B(x-1)}{(x-1)(x+4)} =$$

$$= \left\{ \begin{matrix} A + B = 5 \\ 4A - B = 0 \end{matrix} \right. \Rightarrow \left\{ \begin{matrix} A = 1 \\ B = 4 \end{matrix} \right\} = \frac{1}{x-1} + \frac{4}{x+4}$$

b)

$$\frac{5}{(x-1)(x+4)} = \frac{A}{x-1} + \frac{B}{x+4} = \frac{A(x+4) + B(x-1)}{(x-1)(x+4)} =$$

$$= \left\{ \begin{matrix} A + B = 0 \\ 4A - B = 5 \end{matrix} \right. \Rightarrow \left\{ \begin{matrix} A = 1 \\ B = -1 \end{matrix} \right\} = \frac{1}{x-1} - \frac{1}{x+4}$$

13. a)

$$\int \frac{x}{x+4} dx = \int \frac{x+4-4}{x+4} dx = \int \frac{x+4}{x+4} - \frac{4}{x+4} dx =$$

$$= \int 1 - \frac{4}{x+4} dx = x - 4 \ln|x+4| + C$$

b)

$$\int \frac{x+2}{x+4} dx = \int \frac{x+4-2}{x+4} dx = \int \frac{x+4}{x+4} - \frac{2}{x+4} dx =$$

$$= \int 1 - \frac{2}{x+4} dx = x - 2 \ln|x+4| + C$$

14. a)

$$\int_0^{0.5} 2xe^{2x} dx = \left\{ \begin{matrix} f = x & f' = 1 \\ g' = e^{2x} & g = \frac{1}{2} e^{2x} \end{matrix} \right\} = 2 \left[x \frac{1}{2} e^{2x} \right]_0^{0.5} - 2 \int_0^{0.5} \frac{1}{2} e^{2x} dx =$$

$$= \frac{e}{2} - \frac{1}{2} [e^{2x}]_0^{0.5} = \frac{e}{2} - \frac{e}{2} + \frac{1}{2} = \frac{1}{2}$$

b)

$$\int_0^4 \frac{2x}{\sqrt{x^2 + 9}} dx = \left[2\sqrt{x^2 + 9} \right]_0^4 = 2 \left(\sqrt{4^2 + 9} - \sqrt{0^2 + 9} \right) =$$

$$= 2(5 - 3) = 4$$

15.

$$V = \int_{-\infty}^{\infty} \pi e^{-2|x|}dx = 2\pi \lim_{w\to\infty} \int_0^w e^{-2x}dx = 2\pi \lim_{w\to\infty} \left[\frac{1}{2}e^{-2x}\right]_w^0 = \pi \text{ v.e.}$$

16.

$$y = \frac{1}{2}\ln x - \frac{x^2}{4} \Rightarrow y' = \frac{1}{2x} - \frac{x}{2} \Rightarrow (y')^2 = \frac{1}{4x^2} - \frac{1}{2} + \frac{x^2}{4}$$

$$\int_1^e \sqrt{1+(y')^2}\,dx = \int_1^e \sqrt{\frac{1}{4x^2} + \frac{1}{2} + \frac{x^2}{4}}\,dx = \int_1^e \sqrt{\left(\frac{1}{2x} + \frac{x}{2}\right)^2}\,dx =$$

$$= \frac{1}{2}\int_1^e \left(\frac{1}{x} + x\right)dx = \frac{1}{2}\left[\ln x + \frac{x^2}{2}\right]_1^e = \frac{1+e^2}{4}$$

17.

$$42\frac{\text{cm}^2}{\text{min}} = \frac{dA}{dt} = \frac{dA}{dt}\frac{dr}{dr} = \frac{dA}{dr}\frac{dr}{dt} = \frac{d}{dr}(4\pi r^2)\frac{dr}{dt} = 8\pi r\frac{dr}{dt} = 42 \text{ cm}^2/\text{min}$$

$$\frac{dV}{dt} = \frac{dV}{dt}\frac{dr}{dr} = \frac{dV}{dr}\frac{dr}{dt} = \frac{d}{dr}\left(\frac{4\pi r^3}{3}\right)\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$$

Men då:

$$\frac{dr}{dt} = \frac{42}{8\pi r} \Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} = 4\pi r^2\frac{42}{8\pi r} = 21r = 252 \text{ cm}^3/\text{min}$$

18.

$$y = \frac{x^3}{12} + \frac{1}{x} \Rightarrow y' = \frac{x^2}{4} - \frac{1}{x^2} \Rightarrow (y')^2 = \frac{x^4}{16} - \frac{1}{2} + \frac{1}{x^4}$$

$$S = \int_1^4 \sqrt{1 + \frac{x^4}{16} - \frac{1}{2} + \frac{1}{x^4}}\,dx = \int_1^4 \sqrt{\frac{x^4}{16} + \frac{1}{2} + \frac{1}{x^4}}\,dx =$$

$$= \int_1^4 \sqrt{\left(\frac{x^2}{4} + \frac{1}{x^2}\right)^2}\,dx = \int_1^4 \frac{x^2}{4} + \frac{1}{x^2}\,dx = \left[\frac{x^3}{12} - \frac{1}{x}\right]_1^4 = \frac{4^3}{12} - \frac{1}{4} - \left(\frac{1}{12} - \frac{1}{1}\right) =$$

$$= \frac{64}{12} - \frac{3}{12} - \frac{1}{12} + \frac{12}{12} = \frac{72}{12} = 6 \text{ l.e.}$$

Omfångsrika uppgifter

(sid 190 ff)

1.

A: $\int_0^1 e^{-x}dx = [e^{-x}]_1^0 = 1 - e^{-1} \approx 0.63$

B: $\int_0^1 \frac{e^x}{1+e^x}dx = [\ln(1+e^x)]_0^1 = \ln(1+e) - \ln(1+1) = \ln\frac{1+e}{2} \approx 0.62$

C: $\int_{-1}^0 e^x\cos(e^x)dx = [\sin(e^x)]_{-1}^0 = \sin(1) - \sin(e^{-1}) \approx 0.48$

2. $x^2 + y^2 = 25, \frac{d}{dx} \Rightarrow 2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} = -\frac{4}{3}$ i $(4,3)$

För att hitta var tangenten skär x-axeln kan m hittas:

$y = -\frac{4}{3}x + m \Rightarrow 3 = -\frac{4}{3}4 + m \Rightarrow m = 3 + \frac{16}{3} \Rightarrow y = -\frac{4}{3}x + \frac{25}{3}$

$y = 0 \Rightarrow x = \frac{25}{4} = 6.25 \Rightarrow A = A_\triangle - A_\star = \frac{6.25 \cdot 3}{2} - \pi 5^2 \frac{1}{2\pi} \arctan\left(\frac{3}{4}\right) \approx 1.33$ a. e.

Eller, kanske enklare, är att inse att de båda trianglarna är kongruenta 3, 4, 5-trianglar. Detta gör att den stora triangelns 3-sida sammanfaller med den lilla triangelns 4 sida. Detta ger samma svar som ovan. Och man slipper den implicita deriveringen!

18.

$$\frac{dy}{y(N-y)} = kdt \Rightarrow dy\left(\frac{A}{y} + \frac{B}{N-y}\right) = kdt \Rightarrow dy\frac{A(N-y)+By}{y(N-y)} = kdt = \begin{Bmatrix} A = \dfrac{1}{N} \\ B = \dfrac{1}{N} \end{Bmatrix} \Rightarrow$$

$$\frac{1}{N}dy\left(\frac{1}{y} + \frac{1}{N-y}\right) = kdt \Rightarrow \{\text{integrera båda sidor}\} \Rightarrow \frac{1}{N}\left(\ln y - \ln(N-y)\right) = kt + C$$

$$\ln\frac{y}{N-y} = N(kt+C) \Rightarrow \frac{y}{N-y} = e^{N(kt+C)} \Rightarrow y = Ne^{N(kt+C)} - ye^{N(kt+C)} \Rightarrow$$

$$y(t) = \frac{Ne^{N(kt+C)}}{1 + e^{N(kt+C)}} = \frac{Ne^{Nkt}e^{NC}}{1 + e^{Nkt}e^{NC}} = \frac{Ne^{Nkt}}{C_1 + e^{Nkt}}$$

19. a) $x^3 + y^3 = 3xy$, $\frac{d}{dx}(x^3 + y^3 = 3xy) \Rightarrow 3x^2 + 3y^2\frac{dy}{dx} = 3y + 3x\frac{dy}{dy}$

$$\frac{dy}{dx}(x - y^2) = x^2 - y \Rightarrow \frac{dy}{dx} = \frac{x^2 - y}{x - y^2}$$

$$\frac{dy}{dx} = 0 \text{ då } x^2 = y \Rightarrow x^3 + x^6 = 3x^3 \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \text{och} \begin{cases} x = \sqrt[3]{2} \\ y = \sqrt[3]{4} \end{cases}$$

b) $\frac{dy}{dx} = \infty$ då $x = y^2$ dvs $y^6 + y^3 = 3y^3 \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \text{och} \begin{cases} x = \sqrt[3]{4} \\ y = \sqrt[3]{2} \end{cases}$

c) $x^3 + y^3 = 3xy \Rightarrow x^3 + t^3x^3 = 3xtx \Rightarrow x = \frac{3t}{1+t^3}, y = \frac{3t^2}{1+t^3}, t \neq -1$

20. Produkten av avstånden till $(-a, 0)$ och $(a, 0)$ fås som:

$$\sqrt{(x - (-a))^2 + y^2} \cdot \sqrt{(x-a)^2 + y^2} = a^2 \Rightarrow$$

$$((x+a)^2 + y^2)((x-a)^2 + y^2) = a^4 \Rightarrow$$

$$(x^2 + 2xa + a^2 + y^2)(x^2 - 2xa + a^2 + y^2) = a^4 \Rightarrow$$

$$x^4 - x^3 2a + x^2 a^2 + x^2 y^2 +$$
$$2ax^3 - 4x^2 a^2 + 2xa^3 + 2xay^2 +$$
$$a^2 x^2 - 2xa^3 + a^4 + a^2 y^2 +$$
$$y^2 x^2 - 2xay^2 + a^2 y^2 + y^4 = a^4$$

$$x^4 + 2x^2 y^2 + y^4 = 2x^2 a^2 - 2a^2 y^2 \Rightarrow$$

$$(x^2 + y^2)^2 = 2a^2(x^2 - y^2) \text{ VSV}$$

b)

$$\frac{d}{dx}(x^2 + y^2)^2 = \frac{d}{dx} 2a^2(x^2 - y^2)$$

$$2(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right) = 2a^2\left(2x - 2y\frac{dy}{dx}\right)$$

$$x(x^2 + y^2) + y(x^2 + y^2)\frac{dy}{dx} = a^2 x - a^2 y\frac{dy}{dx}$$

$$y(x^2 + y^2)\frac{dy}{dx} + a^2 y\frac{dy}{dx} = a^2 x - x(x^2 + y^2)$$

$$\frac{dy}{dx}\left(a^2 y + y(x^2 + y^2)\right) = a^2 x - x(x^2 + y^2)$$

$$\frac{dy}{dx} = 0 \text{ då } a^2 x = x(x^2 + y^2) \Rightarrow x^2 + y^2 = a^2$$

Uttrycket är en cirkel med mittpunkt i origo och radie a, dvs 4.

$$x^2 = 4^2 - 2^2 \Rightarrow \begin{cases} x = \pm 2\sqrt{3} \\ y = \pm 2 \end{cases}$$

21. Personen befinner sig på avståndet $\sqrt{20}$ km från fyren. Käglan roterar ett varv på 20 s. Käglans hastighet torde bli:

$$v = \frac{2\pi\sqrt{20}}{20} \approx 1.4 \text{ km/s}$$

(Facit ger 3.1 km/s, oklart varför.)

22. Om papprets kortsida sätts till 1 l.e. blir långsidan $\sqrt{2}$ l.e.. Kallas den skuggade triangelns vertikala katet $1 - y$ fås:

$$(1 - y)^2 + x^2 = y^2 \Leftrightarrow 1 - 2y + y^2 + x^2 = y^2 \Leftrightarrow y = \frac{1}{2}(x^2 + 1)$$
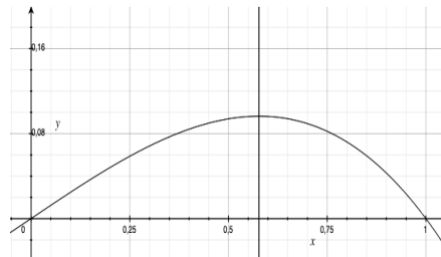
Triangelns area kan uttryckas:

$$A(x) = \frac{1}{2}x(1 - y) = \frac{1}{2}x\left(1 - \frac{1}{2}(x^2 + 1)\right) = \frac{1}{2}x\left(\frac{1}{2} - \frac{x^2}{2}\right) = \frac{1}{4}(x - x^3)$$

Med hjälp av derivatan kan den största arean sökas:

$$\frac{dA(x)}{dx} = \frac{1}{4}(1 - 3x^2) = 0 \text{ då } x = \frac{1}{\sqrt{3}} \approx 0.58$$
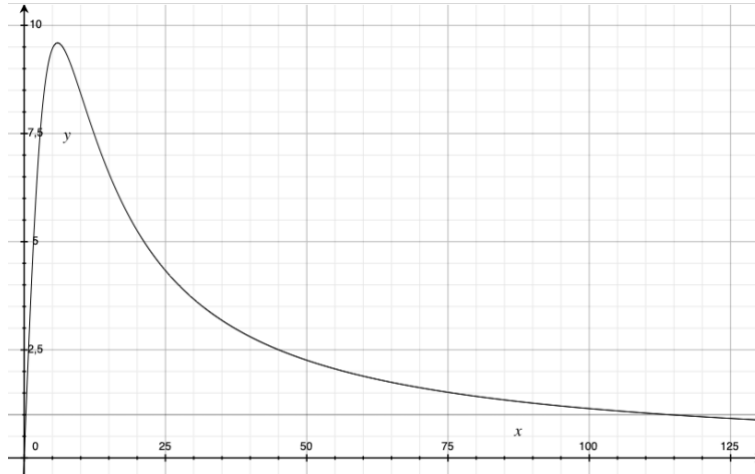
Dvs

$$A_{max} = \frac{1}{4}x(1 - x^2) = \frac{1}{4}\frac{1}{\sqrt{3}}\left(1 - \frac{1}{3}\right) = \frac{1}{6\sqrt{3}} \approx 0.096 \text{ a. e.}$$
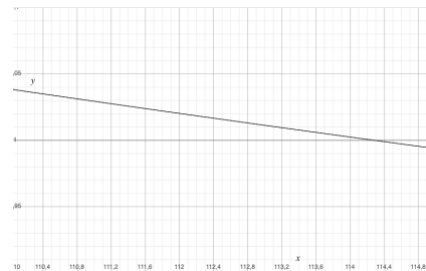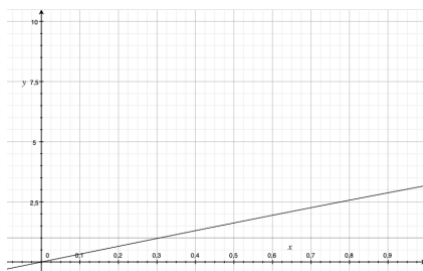
23. Funktionen $v(d)$ visar vinkelns beroende av avståndet d.

a)

$$v(d) = \arctan\frac{7}{d} - \arctan\frac{5}{d}$$

b) Detaljstudier gällande intervallen $0 < d < 1$ m och $110 < d < 115$ m nedan:

Tillsammans ger dessa intervallet $0.3 < d < 114$ m.

c) Cirka 5-6 meter.

d)

$$v'(d) = -\frac{7}{d^2}\frac{1}{1+\left(\frac{7}{d}\right)^2} + \frac{5}{d^2}\frac{1}{1+\left(\frac{5}{d}\right)^2} = 0$$

$$\frac{7}{d^2}\frac{1}{1+\left(\frac{7}{d}\right)^2} = \frac{5}{d^2}\frac{1}{1+\left(\frac{5}{d}\right)^2} \Rightarrow 7\left(1+\left(\frac{5}{d}\right)^2\right) = 5\left(1+\left(\frac{7}{d}\right)^2\right) \Rightarrow$$

$$7\left(1+\frac{25}{d^2}\right) = 5\left(1+\frac{49}{d^2}\right) \Rightarrow 7+\frac{175}{d^2} = 5+\frac{245}{d^2} \Rightarrow 2 = \frac{70}{d^2} \Rightarrow d = \sqrt{35} \approx 5.9 \text{ m}$$

24.

$$\frac{\int_{-2}^{0} x^3 - x^2 - 4x + 3 - (-x^2 + 3)dx}{\int_{0}^{2} -x^2 + 3 - (x^3 - x^2 - 4x + 3)\, dx} = \frac{\int_{-2}^{0} x^3 - 4x\, dx}{-\int_{0}^{2} x^3 - 4x\, dx} = \frac{\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0}}{\left[\frac{x^4}{4} - 2x^2\right]_{2}^{0}} =$$

$$= \frac{\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0}}{\left[\frac{x^4}{4} - 2x^2\right]_{2}^{0}} = \frac{4}{4} = 1$$

27. a)

$$\frac{d}{dx}\cosh x = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x + e^{-x}}{2} = \sinh x$$

$$\frac{d}{dx}\sinh x = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) = \frac{e^x - e^{-x}}{2} = \cosh x$$

$$\frac{d}{dx}\tanh x = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right) = \frac{e^x - e^{-x}}{e^x - e^{-x}} - \frac{e^x + e^{-x}}{(e^x - e^{-x})^2}(e^x + e^{-x}) =$$

$$= \frac{e^{2x} - 2 + e^{-2x}}{(e^x - e^{-x})^2} - \frac{e^{2x} + 2 + e^{-2x}}{(e^x - e^{-x})^2} = -\frac{2^2}{(e^x - e^{-x})^2} = -\frac{1}{\sinh^2 x}$$

En intressant integral, Gaussklockan:

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx$$

Låt oss se om vi kan hitta I^2:

$$I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2} dx \right)^2 = \int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-y^2} dy \, e^{-x^2} dx =$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(y^2+x^2)} dy \, dx = \left\{ \begin{matrix} dydx = \rho d\varphi d\rho \\ y^2 + x^2 = \rho^2 \\ 0 \leq \varphi \leq 2\pi \\ 0 \leq \rho \leq \infty \end{matrix} \right\} = \int_{0}^{\infty} \int_{0}^{2\pi} e^{-\rho^2} \rho d\rho \, d\varphi =$$

$$= 2\pi \frac{1}{2} \left[e^{-\rho^2} \right]_{\infty}^{0} = \pi \Rightarrow I = \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Talkodning:

$$s_w(n) = s(n) + \sum_{i=1}^{10} a_i \gamma_1^i s(n-i) - \sum_{i=1}^{10} a_i \gamma_2^i s_w(n-i), \quad n = 0 \cdots L-1$$

$$s_w(n) = s(n) + a_1 \gamma_1 s(n-1) + \cdots + a_{10} \gamma_1^{10} s(n-10)$$

$$- \left(a_1 \gamma_2 s_w(n-1) + \cdots + a_{10} \gamma_2^{10} s_w(n-10) \right)$$

där , $n = 0 \cdots 159$

eller:

$$s_w(n) = s(n) + a_1 0.9_1 s(n-1) + \cdots + a_{10} 0.9^{10} s(n-10)$$

$$- \left(a_1 0.6 s_w(n-1) + \cdots + a_{10} 0.6^{10} s_w(n-10) \right)$$

$$D = \sum_{n=1}^{40} \left(x(n) - gx(n-L) \right)^2 = \sum x(n)^2 - 2g \sum x(n)x(n-L) + \sum g^2 x^2(n-L)$$

$$\frac{\partial D}{\partial g} = -2 \sum x(n)x(n-L) + 2g \sum x^2(n-L) = 0$$

$$g = \frac{\sum x(n)x(n-L)}{\sum x^2(n-L)}$$

$$D = \sum x^2(n) - 2g \sum x(n)x(n-L) + \sum g^2 x^2(n-L) =$$

$$= \sum x^2(n) - 2 \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} + \left[\frac{\sum x(n)x(n-L)}{\sum x^2(n-L)} \right]^2 \sum x^2(n-L) =$$

$$= \sum x^2(n) - 2 \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} + \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} =$$

$$= \sum x^2(n) - \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)}$$

Testa $41 \leq L \leq 169$