Matematik 5 svar

Kapitel 4

4101. a) $y = (2x+3)^4 \Rightarrow y'(x) = 4(2x+3)^3 2 = 8(2x+3)^3$

b) $y = (1-4x)^5 \Rightarrow y'(x) = -5 \cdot 4(1-4x)^4 = -20(1-4x)^4$

c) $y = (1+2x^3)^4 \Rightarrow y'(x) = 4 \cdot 2 \cdot 3x^2(1+2x^3)^3 = 24x^2(1+2x^3)^3$

d) $y = (x^2+2x)^3 \Rightarrow y'(x) = 3(x^2+2x)^2(2x+2) = 6(x+1)(x^2+2x)^2$

e) $y = \left(1+\frac{1}{2x^3}\right)^4 = \left(1+\frac{1}{2}x^{-3}\right)^4 \Rightarrow y'(x) = 4\left(1+\frac{1}{2x^3}\right)^3 \frac{-3}{2}\, x^{-4} = -\frac{6}{x^4}\left(1+\frac{1}{2x^3}\right)^3$

f) $y = (\sin x + \cos x)^3 \Rightarrow y'(x) = 3(\cos x - \sin x)(\sin x + \cos x)^2$

4102. a) $f(x) = (3x+5)^{20} \Rightarrow f'(x) = 20(3x+5)^{19}3x = 60x(3x+5)^{19}$

b) $g(x) = \sqrt{3x-5} \Rightarrow g'(x) = \frac{3}{2\sqrt{3x-5}}$

c) $h(x) = \frac{1}{(2x+1)^2} = (2x+1)^{-2} \Rightarrow h'(x) = -2 \cdot 2(2x+1)^{-3} = -\frac{4}{(2x+1)^3}$

d) $f(t) = (e^{3t}+t)^4 \Rightarrow f'(t) = 4(3e^{3t}+1)(e^{3t}+t)^3$

e) $g(u) = \frac{2}{e^{3u}} = 2e^{-3u} \Rightarrow g'(u) = -6e^{-3u} = -\frac{6}{e^{3u}}$

f) $h(v) = \frac{1}{\sqrt{v^2+3}} = (v^2+3)^{-\frac{1}{2}} \Rightarrow h'(x) = -\frac{1}{2}2v(v^2+3)^{-\frac{3}{2}} = -v(v^2+3)^{-\frac{3}{2}} = \frac{-v}{(v^2+3)^{1.5}}$

4103. a) $f(x) = e^{x^2} \Rightarrow f'(x) = 2xe^{x^2}$

b) $f(v) = \sin^2 v \Rightarrow f'(x) = 2\sin v \cos v = \sin 2v$

4104. a) $y = (\sin x + x)^2 \Rightarrow y'(x) = 2(\sin x + x)(\cos x + 1)$

b) $y = (2x+e^{2x})^4 \Rightarrow y'(x) = 4(2x+e^{2x})^3(2+2e^{2x})$

c) $y = e^{(x^2+1)^2} \Rightarrow y'(x) = 2(x^2+1)2xe^{(x^2+1)^2} = 4x(x^2+1)e^{(x^2+1)^2}$

d)

$$y = \ln(x + 2x^3) \Rightarrow y'(x) = \frac{1 + 6x^2}{x + 2x^3}$$

4105. A: $f(x) = \ln e^{3x} = 3x \ln e = 3x \Rightarrow f'(x) = 3$

B:

$$f(x) = \ln(\cos 3x) \Rightarrow f'(x) = -\frac{3 \sin 3x}{\cos 3x} = -3 \tan 3x$$

4106. a)

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = 5x^2$$

b)

$$\frac{dz}{dx} = \frac{dy}{dx} \cdot \frac{dz}{dy} = \frac{dy}{dx} \cdot \left(\frac{dy}{dz}\right)^{-1} = 8 \cdot \frac{1}{2} = 4$$

4107. a) $y = x^3(x^2 - 1)^3 \Rightarrow y' = 3x^2(x^2 - 1)^3 + x^3 3(x^2 - 1)^2 2x =$

$$= 3x^2(x^2 - 1)^3 + 6x^4(x^2 - 1)^2 = 3x^2(x^2 - 1)^2(3x^2 - 1)$$

b) $y = (1 + x^2)(x^2 - 1)^3 \Rightarrow y' = 2x(x^2 - 1)^3 + (1 + x^2)3 \cdot 2x(x^2 - 1)^2 =$

$$= 2x(x^2 - 1)^2[x^2 - 1 + (1 + x^2)3] = 2x(x^2 - 1)^2[x^2 - 1 + 3 + 3x^2] =$$

$$= 4x(x^2 - 1)^2(2x^2 + 1)$$

c) $= e^{x^2}(x^3 - 1)^2 \Rightarrow y' = 2xe^{x^2}(x^3 - 1)^2 + e^{x^2} 2 \cdot 3x^2(x^3 - 1)=$

$$= 2xe^{x^2}(x^3 - 1)(x^3 - 1 + 3x)$$

d) $y = \sin 2x(\cos 4x + 2) \Rightarrow y' = 2\cos 2x(\cos 4x + 2) - 4\sin 2x \sin 4x =$

$$= 2\cos 2x(\cos 4x + 2) - 4\sin 2x \cdot 2\sin 2x \cos 2x = 2\cos 2x(\cos 4x + 2 - 4\sin^2 2x)$$

4108. $\frac{ds}{dt} = 0.3 \frac{\text{cm}}{\text{h}},\ V = s^3 \Rightarrow \frac{dV}{dS} = 3s^2$ *och* $\frac{dV}{dt} = \frac{dV}{dt}\frac{ds}{ds} = \frac{dV}{ds}\frac{ds}{dt} = 3s^2 0.3 \approx 5.1\ \text{dm}^3/\text{h}$

4109. $A = x^2, \frac{dA}{dt} = 2x\frac{ds}{ds} = 2 \cdot 15 \cdot 3.7 = 111 \frac{cm^2}{s} \approx 1.1\ \text{dm}^2/\text{s}$

4110. $50\ \text{m}^2 \cdot \frac{dx}{dt} = 0.09 \frac{\text{m}^3}{\text{min}} \Rightarrow \frac{dx}{dt} = \frac{0.09}{50}\frac{\text{m}^3}{\text{min}}\frac{1}{\text{m}^2} = 1.8\ \frac{\text{mm}}{\text{min}}$

4111. $V = A \cdot h \Rightarrow \frac{dV}{dt} = A\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{A}\frac{dV}{dt} = \frac{1}{\pi 1.2^2} 0.065 \frac{\text{m}}{\text{min}} \approx 1.4\ \text{cm/min}$

4112. $V = \frac{4\pi r^3}{3}, \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt} = \frac{1}{4\pi 25^2} 1600 \approx 2\ \text{mm/s}$

4113. $\frac{h}{r} = 1.5, \frac{dh}{dt} = \frac{dh}{dt}\frac{dV}{dV} = \frac{dV}{dt}\frac{dh}{dV} = \frac{dV}{dt}\left(\frac{dV}{dh}\right)^{-1}$ men $V = \frac{bh}{3} = \frac{\pi r^2 h}{3} = \frac{\pi\left(\frac{h}{1.5}\right)^2 h}{3} = \frac{\pi h^3}{6.75}$

$$\frac{dV}{dh}=\frac{\pi 3h^2}{6.75}=\frac{\pi h^2}{2.25}\Rightarrow\frac{dh}{dt}=\frac{dV}{dt}\left(\frac{dV}{dh}\right)^{-1}=2\cdot\frac{2.25}{\pi 2^2}\approx 3.6\text{ cm/min}$$

4114. $A=\pi r^2,\frac{dA}{dt}=\frac{dA}{dt}\frac{dr}{dr}=\frac{dA}{dr}\frac{dr}{dt}=\pi 2r\cdot\frac{dr}{dt}=\pi 2\cdot(4\cdot 1.2)\cdot 1.2\approx 36\text{ dm}^2/\text{s}$

4115.

$$\frac{1}{2}=\frac{h}{d}=\frac{h}{2r}\Rightarrow h=r$$

$$\frac{dh}{dt}=\frac{dh}{dt}\frac{dV}{dV}=\frac{dV}{dt}\frac{dh}{dV}=\frac{dV}{dt}\left(\frac{dV}{dh}\right)^{-1}\text{ men }V=\frac{bh}{3}=\frac{\pi r^2h}{3}=\frac{\pi h^2h}{3}=\frac{\pi h^3}{3}$$

$$\frac{dV}{dh}=\frac{\pi 3h^2}{3}=\pi h^2\Rightarrow\frac{dh}{dt}=0.4\cdot\frac{1}{\pi 2^2}=\frac{0.1}{\pi}\approx 3.2\text{ cm/min}$$

4116. a)

$$\frac{dV}{dt}=\frac{dV}{dr}\frac{dr}{dt}\Rightarrow\frac{dr}{dt}=\frac{dV}{dt}\left(\frac{dV}{dr}\right)^{-1}=\frac{dV}{dt}\left(\frac{d}{dr}\frac{4\pi r^3}{3}\right)^{-1}\Rightarrow$$
$$\frac{dr}{dt}=\frac{dV}{dt}(4\pi r^2)^{-1}=\frac{150}{4\pi 25^2}\approx 0.19\text{ mm/min}$$

b)

$$\frac{dA}{dt}=\frac{dA}{dr}\frac{dr}{dt}=\frac{dr}{dt}\left(\frac{d}{dr}4\pi r^2\right)=\frac{150}{4\pi 25^2}\cdot 8\pi 25=12\text{ cm}^2/\text{min}$$

4117. Konens volym finner man som:

$$V_{\text{kon}}=\frac{bh}{3}=\frac{\pi r^2h}{3}=\{\text{toppvinkel}=90°\Rightarrow r=h\}=\frac{\pi h^3}{3}$$

$$\frac{dV}{dh}=\pi h^2\text{ dvs }\frac{dV}{dt}=\frac{dV}{dh}\frac{dh}{dt}\Rightarrow\frac{dh}{dt}=\left(\frac{dV}{dh}\right)^{-1}\frac{dV}{dt}=\frac{1}{\pi 3^2}2.5\approx 8.8\text{ mm/min}$$

4118.

$$V=\frac{4\pi r^3}{3},A=4\pi r^2,\frac{dV}{dt}=\frac{dV}{dr}\frac{dr}{dt}=4\pi r^2\frac{dr}{dt}\Rightarrow\frac{dr}{dt}\text{ är konstant}$$

4119. $y^2=2x$ {implicit derivering ger}$2y\frac{dy}{dx}=2\Rightarrow\frac{dy}{dx}=\frac{1}{y}\Rightarrow y'(2)=-\frac{1}{2}$

4120. a) $y^2=9x$ {implicit derivering ger} $2y\frac{dy}{dx}=9\Rightarrow\frac{dy}{dx}=\frac{9}{2y}\Rightarrow y'(4)=\frac{9}{12}=\frac{3}{4}$

b) Enpunktsformeln ger $y-6=\frac{3}{4}(x-4)\Rightarrow y=\frac{3}{4}x+6-4\frac{3}{4}=\frac{3}{4}x+3$

c) $y=-\frac{3}{4}x-3$ (fel i facit)

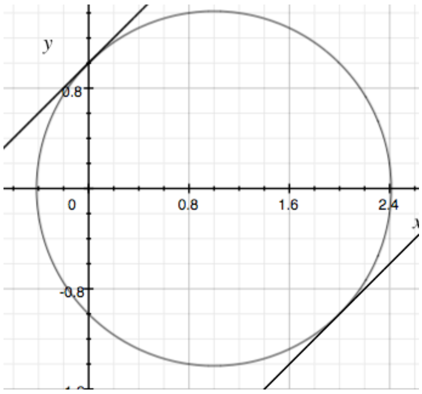
4121. a) $x^2+2y^2=1$ {implicit derivering ger} $2x+4y\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\frac{x}{2y}$

b) $x^2+2y=y^3$ {implicit derivering ger} $2x+2\frac{dy}{dx}=3y^2\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{2x}{3y^2-2}$

4128.

$$(x-1)^2+y^2=2 \quad \frac{d}{dx} \Rightarrow 2(x-1)+2y\frac{dy}{dx}=0 \text{ dvs då } 1-x=y$$

$$y^2+y^2=2 \Rightarrow (x_1,y_1)=(0{,}1)\ och\ (x_2,y_2)=(2,-1)$$



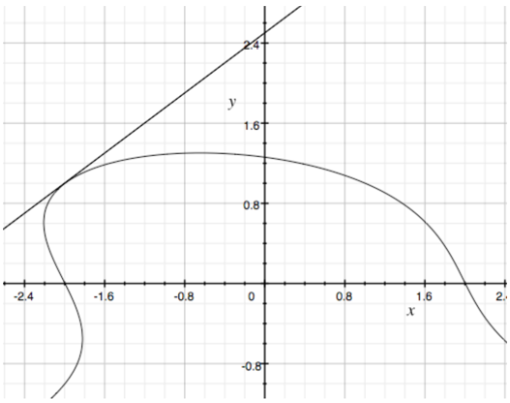
4129.

$$x^2+xy+2y^3=4 \quad \frac{d}{dx} \Rightarrow 2x+y+x\frac{dy}{dx}+2\cdot 3y^2\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(x+6y^2)=-2x-y \Rightarrow \frac{dy}{dx}=-\frac{2x+y}{x+6y^2}=-\frac{-4+1}{-2+6}=\frac{3}{4}$$

$$y=kx+m \Rightarrow 1=\frac{3}{4}(-2)+m \Rightarrow m=1+\frac{3}{2}=2.5$$

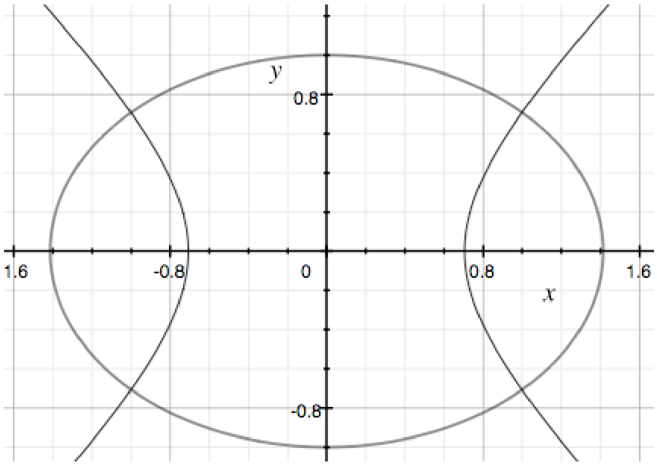
$$y=0.75x+2.5$$



4130.

$$\begin{cases} x^2 + 2y^2 = 2 \\ 2x^2 - 2y^2 = 1 \end{cases} \Rightarrow 3x^2 = 3 \Rightarrow \begin{cases} x = \pm 1 \\ y = \pm \dfrac{1}{\sqrt{2}} \end{cases}$$

$$\frac{d}{dx}\begin{cases} x^2 + 2y^2 = 2 \\ 2x^2 - 2y^2 = 1 \end{cases} \Rightarrow \begin{cases} 2x + 4y\dfrac{dy}{dx} = 0 \\ 4x - 4y\dfrac{dy}{dx} = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{dy}{dx} = -\dfrac{x}{2y} = -\dfrac{1}{\sqrt{2}} \\ \dfrac{dy}{dx} = \dfrac{x}{y} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2} \end{cases} \quad \text{VSV}$$



4131.

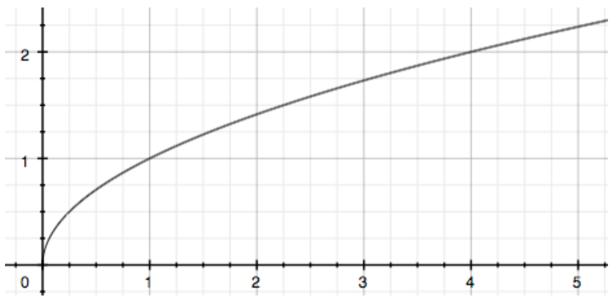
$$\frac{d}{dt}(y^2 = 300^2 + x^2) \Rightarrow 2y\frac{dy}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{x}{\sqrt{300^2 + x^2}} 50 \text{ m/min}$$

a) $\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{300}{\sqrt{300^2+300^2}} 50 \approx 35 \text{ m/min}$

b) $\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt} = \frac{400}{\sqrt{300^2+400^2}} 50 = 40 \text{ m/min}$

4132.

$$\frac{d}{dt}(y^2 = x) \Rightarrow 2y\frac{dy}{dt} = \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{1}{2y}\frac{dx}{dt} = \frac{1.5}{4} = \frac{3}{8} \approx 0.38 \text{ m/s}$$



4133.

$$x^2 = 1500^2 + y^2, \qquad \frac{d}{dt}(x^2 = 1500^2 + y^2) \Rightarrow 2x\frac{dx}{dt} = 2y\frac{dy}{dt} \Rightarrow$$

$$\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt} = \frac{1000}{\sqrt{1500^2 + 1000^2}} 200 \approx 110 \text{ m/s}$$

4134.

$$\frac{d}{dt}(y^2 = 5^2 - x^2) \Rightarrow 2y\frac{dy}{dt} = -2x\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{3}{\sqrt{5^2 - 3^2}}0.04 = -0.03 \text{ m/s}$$

4135.

$$v = 540\frac{\text{km}}{\text{h}} = 150 \text{ m/s}$$

$$a(t) = \sqrt{(vt)^2 + h^2} \Rightarrow \frac{da}{dt} = \frac{1}{2}\frac{2tv^2}{\sqrt{(vt)^2 + h^2}} = \{t = 60 \text{ s}\} \approx 130\frac{\text{m}}{\text{s}} \approx 470 \text{ km/h}$$

4201. a)

$$\int_0^\infty e^{-x}dx = \lim_{t\to\infty}\int_0^t e^{-x}dx = \lim_{t\to\infty}[e^{-x}]_t^0 = 1$$

b)

$$\int_0^\infty e^{-2x}dx = \lim_{t\to\infty}\int_0^t e^{-2x}dx = \frac{1}{2}\lim_{t\to\infty}[e^{-2x}]_t^0 = \frac{1}{2}$$

c)

$$\int_0^\infty \sin x\,dx = \lim_{t\to\infty}\int_0^t \sin x\,dx = \lim_{t\to\infty}[\cos x]_0^t \Rightarrow \text{divergent}$$

4202. a)

$$\int_1^\infty \frac{1}{\sqrt{x}}dx = \left[2\sqrt{x}\right]_1^\infty \Rightarrow \text{divergent}$$

b)

$$\int_1^\infty \frac{1}{x}dx = [\ln x]_1^\infty \Rightarrow \text{divergent}$$

c)

$$\int_1^\infty \frac{1}{x^2}dx = \left[\frac{1}{x}\right]_\infty^1 = 1$$

4203.

$$A = \int_0^\infty e^{-0.2x}dx = [5e^{-0.2x}]_\infty^0 = 5 \text{ a. e.}$$

4204. a)

$$\lim_{w\to\infty}\int_2^w \frac{2}{x^5}\,dx = \lim_{w\to\infty}\left[-\frac{2}{4x^4}\right]_2^w = \frac{1}{2}\lim_{w\to\infty}\left[\frac{1}{2^4}-\frac{1}{w^4}\right] = \frac{1}{32}$$

b)

$$\lim_{w\to\infty}\int_0^w \frac{4}{(2x+1)^3}\,dx = \lim_{w\to\infty}\left[-\frac{1}{2}\cdot\frac{1}{2}\frac{4}{(2x+1)^2}\right]_0^w = \lim_{w\to\infty}\left[1-\frac{1}{(2w+1)^2}\right] = 1$$

c)

$$\lim_{w\to\infty}\int_0^w 3e^{-2x}\,dx = \lim_{w\to\infty}\left[-\frac{3}{2}e^{-2x}\right]_0^w = \frac{3}{2}$$

d)

$$\lim_{w\to\infty}\int_4^w \frac{2}{x\sqrt{x}}\,dx = \lim_{w\to\infty}\left[-2\frac{2}{\sqrt{x}}\right]_4^w = 2$$

e)

$$\lim_{w\to\infty}\int_1^w \frac{2}{\sqrt{x}}\,dx = \lim_{w\to\infty}\left[2\cdot 2\sqrt{x}\right]_1^w \text{ divergerar}$$

f)

$$\lim_{w\to\infty}\int_{-w}^1 e^{2x}\,dx = \lim_{w\to\infty}\left[\frac{1}{2}e^{2x}\right]_{-w}^1 = \frac{e^2}{2}$$

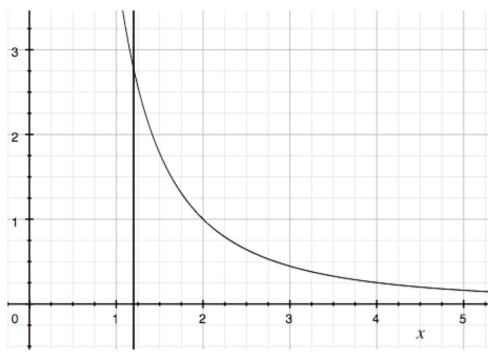
4205.

$$\lim_{a\to\infty}\int_0^a 4e^{-0.9x}\,dx = \lim_{a\to\infty}\left[-\frac{4}{0.9}e^{-0.9x}\right]_0^a = \frac{4}{0.9}$$

4206.

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_0^1 = 2$$

4207.



$$\lim_{w\to\infty}\int_a^w \frac{4}{x^2}\,dx = \lim_{w\to\infty}\left[-\frac{4}{x}\right]_a^w = \frac{4}{a} = 4 \Rightarrow a = 1$$

4208.

$$\lim_{w\to\infty}\int_1^w \frac{1}{x^k}\,dx = \lim_{w\to\infty}\int_1^w x^{-k}\,dx = \lim_{w\to\infty}\left[\frac{1}{-k+1}x^{-k+1}\right]_1^w = \frac{1}{1-k}\lim_{w\to\infty}\left[\frac{1}{x^{k-1}}\right]_1^w \Longrightarrow$$

4209.

$$V = \int_1^5 \pi(2x)^2 dx = 4\pi \int_1^5 x^2 dx = \frac{4\pi}{3}\,[x^3]_1^5 = \frac{4\pi 124}{3} \approx 519 \text{ v. e.}$$

Volymen kallas en stympad kon.

4212.

$$V = \lim_{w\to\infty} \int_0^w \pi e^{-2x} dx = \frac{\pi}{2} \lim_{w\to\infty} [e^{-2x}]_w^0 = \frac{\pi}{2} \text{ v. e.}$$

4213. Volymen är den som i vänsterkant begränsas av $x = 2$.

$$V = \lim_{w\to\infty} \int_2^w \pi \left(\frac{1}{x\sqrt{x}}\right)^2 dx = \pi \lim_{w\to\infty} \int_2^w \frac{1}{x^3} dx = \frac{\pi}{2} \lim_{w\to\infty} \left[\frac{1}{x^2}\right]_w^2 = \frac{\pi}{8} \text{ v. e.}$$

4214. Volymen är den som i vänsterkant begränsas av $x = 2$.

$$V = \lim_{w\to\infty} \int_2^w \frac{\pi dx}{(x-1)^2} = \pi \lim_{w\to\infty} \left[\frac{1}{x-1}\right]_w^2 = \pi \text{ v. e.}$$

4215. a)

$$\lim_{w\to\infty} \int_1^w \frac{1}{x} dx = \lim_{w\to\infty} [\ln|x|]_1^w \text{ divergent } \Rightarrow \text{ oänligt mycket färg!}$$

b)

$$\lim_{w\to\infty} \int_1^w \pi \frac{1}{x^2} dx = \pi \lim_{w\to\infty} \left[\frac{1}{x}\right]_w^1 = \pi \text{ v. e. färg.}$$

4216.

$$\int_{-1}^3 \pi(x+1)dx = \pi \left[\frac{x^2}{2} + x\right]_{-1}^3 = \pi\left(\frac{9}{2} + 3 - \frac{1}{2} + 1\right) = 8\pi \text{ v. e.}$$

4217.

$$\int_{-1}^a \pi(x+1)dx = \pi \left[\frac{x^2}{2} + x\right]_{-1}^a = \pi\left(\frac{a^2}{2} + a - \frac{1}{2} + 1\right) = 16\pi$$

$$\frac{a^2}{2} + a - \frac{1}{2} + 1 = 16 \Rightarrow a^2 + 2a - 31 = 0$$

$$a = -1 \pm \sqrt{1+31} - 1 \pm \sqrt{32} = 4\sqrt{2} - 1 \approx 4.7$$

4218. a)

$$\int_0^2 10e^{-0.5t} dt = 10 \cdot 2\,[e^{-0.5t}]_2^0 = 20(1 - e^{-1}) \approx 12.6 \text{ g}$$

b) 20 g

4219.

$$\lim_{w\to\infty}\int_0^w 100e^{-0.05t}dt = 2000\lim_{w\to\infty}[e^{-0.05t}]_w^0 = 2000 \text{ liter}$$

4220.

$$\lim_{w\to\infty}\int_0^w 2e^{-0.025x}dx = 80\lim_{w\to\infty}[e^{-0.025t}]_w^0 = 80$$

Den ursprungliga temperaturen var 80°.

4221.

$$\lim_{w\to\infty}\int_0^w 0.001e^{-0.001x}dx = \lim_{w\to\infty}[e^{-0.001t}]_w^0 = 1$$

Sannolikheten att den förr eller senare går sönder = 1.

4222. a)

$$5200\int_0^2 \frac{dt}{(1+t)^2} = 5200\left[\frac{1}{1+t}\right]_2^0 = 5200\left(1-\frac{1}{3}\right) \approx 3500 \text{ st}$$

b) 5200 st

4223.

$$\lim_{w\to\infty}\int_0^w 0.001e^{-0.05t}dt = 0.02\lim_{w\to\infty}[e^{-0.05t}]_w^0 = 0.02 \text{ g}$$

Facit svarar μg, troligen skulle det varit μg i uppgiften.

4224.

$$i(t) = 4.41\cdot 10^{-5}\cdot 0.956^t = 4.41\cdot 10^{-5}\cdot e^{t\ln 0.956}$$

$$q_{tot} = 4.41\cdot 10^{-5}\int_0^\infty e^{t\ln 0.956}dt = \frac{4.41\cdot 10^{-5}}{\ln 0.956}\left[e^{t\ln 0.956}\right]_0^\infty =$$

$$= \frac{4.41\cdot 10^{-5}}{\ln 0.956}(-1) = 0.98 \text{ mC}$$

4225. a)

$$\int_{R_j}^R F(r)dr = 6.672\cdot 10^{-11}M\cdot m\int_{R_j}^R \frac{dr}{r^2} = 6.672\cdot 10^{-11}M\cdot m\left[\frac{1}{r}\right]_R^{R_j} =$$

$$= 6.672\cdot 10^{-11}M\cdot m\left(\frac{1}{R_j}-\frac{1}{R}\right) =$$

$$= 6.672\cdot 10^{-11}\cdot 5.9735\cdot 10^{24}\cdot 15\,000\left(\frac{1}{6.37\cdot 10^6}-\frac{1}{6.38\cdot 10^6}\right) \approx 1.47 \text{ GJ}$$

Energin som krävs att lyfta 15 ton 10 km ut från jordens yta.

b) Cirka 940 GJ för att lyfta 15 ton bort från jordens gravitationsfält.

4301. a) $\frac{x^2+3}{x^2} = 1+\frac{3}{x^2}$ b) $\frac{10-x}{2x} = \frac{10}{2x}-\frac{x}{2x} = \frac{5}{x}-\frac{1}{2}$ c) $\frac{x(x+2)+1}{x+2} = \frac{x(x+2)}{x+2}+\frac{1}{x+2} = x+\frac{1}{x+2}$

4302. a) $\frac{x}{x+4}=\frac{x+4-4}{x+4}=\frac{x+4}{x+4}-\frac{4}{x+4}=1-\frac{4}{x+4}$

b) $\frac{x}{x-7}=\frac{x-7+7}{x-7}=\frac{x-7}{x-7}+\frac{7}{x-7}=1+\frac{7}{x-7}$

4303. a) $\frac{x}{x+1}=\frac{x+1-1}{x+1}=\frac{x+1}{x+1}-\frac{1}{x+1}=1-\frac{1}{x+1}$

b) $\frac{x}{x-5}=\frac{x-5+5}{x-5}=\frac{x-5}{x-5}+\frac{5}{x-5}=1+\frac{5}{x-5}$

c) $\frac{x}{x+8}=\frac{x+8-8}{x+8}=\frac{x+8}{x+8}-\frac{8}{x+8}=1-\frac{8}{x+8}$

4304. $\frac{1}{t(t+1)}=\frac{a}{t}+\frac{b}{t+1}=\frac{a(t+1)}{t(t+1)}+\frac{bt}{(t+1)t}=\frac{at+a+bt}{t(t+1)}=\left\{\begin{matrix}a=1\\b=-1\end{matrix}\right\}=\frac{1}{t}-\frac{1}{t+1}$

4305.

$$\frac{3x}{(x+2)(x-1)}=\frac{a}{x+2}+\frac{b}{x-1}=\frac{a(x-1)}{(x+2)(x-1)}+\frac{b(x+2)}{(x+2)(x-1)}=$$

$$=\frac{a(x-1)+b(x+2)}{(x+2)(x-1)}=\left\{\begin{matrix}a+b=3\\-a+2b=0\end{matrix}\right\}\Rightarrow\left\{\begin{matrix}b=1\\a=2\end{matrix}\right.=\frac{2}{x+2}+\frac{1}{x-1}$$

4306. a)

$$\frac{x+1}{x+2}=\frac{x+1+1-1}{x+2}=\frac{x+2-1}{x+2}=\frac{x+2}{x+2}-\frac{1}{x+2}=1-\frac{1}{x+2}$$

b)

$$\frac{x-2}{x+1}=\frac{x-2+3-3}{x+1}=\frac{x+1-3}{x+1}=\frac{x+1}{x+1}-\frac{3}{x+1}=1-\frac{3}{x+1}$$

c)

$$\frac{x+1}{x-3}=\frac{x+1-4+4}{x-3}=\frac{x-3}{x-3}+\frac{4}{x-3}=1+\frac{4}{x-3}$$

4307. a)

$$\frac{x+1}{(x-2)(x-1)}=\frac{a}{x-2}+\frac{b}{x-1}=\frac{a(x-1)}{(x-2)(x-1)}+\frac{b(x-2)}{(x-2)(x-1)}=$$

$$=\frac{a(x-1)+b(x-2)}{(x-2)(x-1)}=\left\{\begin{matrix}a+b=1\\-a-2b=1\end{matrix}\Rightarrow\begin{matrix}b=-2\\a=3\end{matrix}\right\}=$$

$$=\frac{3}{x-2}-\frac{2}{x-1}$$

b)

$$\frac{4}{x(x-4)}=\frac{a}{x}+\frac{b}{x-4}=\frac{a(x-4)}{x(x-4)}+\frac{bx}{x(x-4)}=\frac{a(x-4)+bx}{x(x-4)}=$$

$$=\left\{\begin{matrix}a=-1\\b=1\end{matrix}\right\}=\frac{1}{x-4}-\frac{1}{x}$$

c)

$$\frac{2x+5}{x^2-5x}=\frac{2x+5}{x(x-5)}=\frac{a}{x}+\frac{b}{x-5}=\frac{a(x-5)+bx}{x(x-5)}=\left\{\begin{matrix}a+b=2\\a=-1\end{matrix}\Rightarrow\begin{matrix}a=-1\\b=3\end{matrix}\right\}=$$

$$=\frac{3}{x-5}-\frac{1}{x}$$

4308.

$$\frac{1-x}{(x+2)^2}=\frac{a}{x+2}+\frac{b}{(x+2)^2}=\frac{a(x+2)+b}{(x+2)^2}=\left\{\begin{matrix}a=-1\\b=3\end{matrix}\right\}=\frac{3}{(x+2)^2}-\frac{1}{x+2}$$

4309. a)

$$\frac{x}{(x-1)^2}=\frac{a}{(x-1)^2}+\frac{b}{x-1}=\frac{a+b(x-1)}{(x-1)^2}=\begin{Bmatrix}b=1\\a=1\end{Bmatrix}=\frac{1}{(x-1)^2}+\frac{1}{x-1}$$

b)

$$\frac{x+3}{(x-1)^2}=\frac{a}{(x-1)^2}+\frac{b}{x-1}=\frac{a+b(x-1)}{(x-1)^2}=\begin{Bmatrix}b=1\\a=4\end{Bmatrix}=\frac{4}{(x-1)^2}+\frac{1}{x-1}$$

4310. a)

$$\int_1^3\frac{3x+x^2}{x}dx=\int_1^3 3+xdx=\left[3x+\frac{x^2}{2}\right]_1^3=9+\frac{9}{2}-3-\frac{1}{2}=10$$

b)

$$\int_1^2\frac{2+x}{x}dx=\int_1^2\frac{2}{x}+1=[2\ln x+x]_1^2=2\ln 2+2-2\ln 1-1=2\ln 2+1$$

4311. a)

$$\int\frac{x}{x+2}dx=\int\frac{x+2}{x+2}-\frac{2}{x+2}dx=x-2\ln|x+2|+C$$

b)

$$\int\frac{x}{x-2}dx=\int\frac{x-2}{x-2}+\frac{2}{x-2}dx=x+2\ln|x-2|+C$$

4312. a)

$$\int_2^3\frac{1}{x(x+1)}dx=\left[\frac{A}{x}+\frac{B}{x+1}=\frac{A(x+1)+Bx}{x(x+1)}\Rightarrow\begin{matrix}A=1\\B=-1\end{matrix}\right]=$$

$$=\int_2^3\frac{1}{x}-\frac{1}{x+1}dx=[\ln|x|-\ln|x+1|]_2^3=\ln 3-\ln 4-\ln 2+\ln 3=\ln\frac{9}{8}$$

b)

$$\int_{-1}^0\frac{1}{(x-1)(x-2)}dx=\left[\frac{A}{x-1}+\frac{B}{x-2}=\frac{A(x-2)+B(x-1)}{(x-1)(x-2)}\Rightarrow\begin{matrix}A+B=0\\-2A-B=1\end{matrix}\Rightarrow\begin{matrix}A=-1\\B=1\end{matrix}\right]=$$

$$=\int_{-1}^0\frac{1}{x-2}-\frac{1}{x-1}dx=[\ln|x-2|-\ln|x-1|]_{-1}^0=\ln 2-\ln 1-\ln 3+\ln 2=\ln\frac{4}{3}$$

4313.

$$\int_0^1\frac{x+4}{(x-2)(x+1)}dx=\left[\frac{A}{x-2}+\frac{B}{x+1}=\frac{A(x+1)+B(x-2)}{(x-2)(x+1)}\Rightarrow\begin{matrix}A+B=1\\A-2B=4\end{matrix}\Rightarrow\begin{matrix}A=2\\B=-1\end{matrix}\right]=$$

$$= \int_0^1 \frac{2}{x-2} - \frac{1}{x+1} dx = [2\ln|x-2| - \ln|x+1|]_0^1 = 2\ln 1 - \ln 2 - 2\ln 2 + \ln 1 =$$

$$= -3\ln 2 \approx -2.1$$

4314. a)

$$\int \frac{2}{x+2} dx = 2\ln|x+2| + C$$

b)

$$\int \frac{2}{x^2+2x} dx = \int \frac{2}{x(x+2)} dx = \left[\frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)+Bx}{x(x+2)} \Rightarrow \begin{matrix} A=1 \\ B=-1 \end{matrix}\right] =$$

$$= \int \frac{1}{x} - \frac{1}{x+2} dx = \ln|x| - \ln|x+2| + C$$

c)

$$\int \frac{x}{x+1} dx = \int \frac{x+1}{x+1} - \frac{1}{x+1} dx = x - \ln|x+1| + C$$

d)

$$\int \frac{2x}{x-2} dx = \int \frac{2x-4}{x-2} + \frac{4}{x-2} dx = \int 2 + \frac{4}{x-2} dx = 2x + 4\ln|x-2| + C$$

4315.

$$\int_5^{e+4} \frac{x}{x-4} dx = \int_5^{e+4} \frac{x-4}{x-4} + \frac{4}{x-4} dx = [x + 4\ln|x-4|]_5^{e+4} = e + 4 + 4 - 5 = e + 3$$

4316. a)

$$\int \frac{2x+1}{(x-1)(x+2)} dx = \left[\frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{(x-1)(x+2)} \Rightarrow \begin{matrix} A=1 \\ B=1 \end{matrix}\right] =$$

$$= \int \frac{1}{x-1} + \frac{1}{x+2} dx = \ln|x-1| + \ln|x+2| + C$$

b)

$$\int \frac{2x}{x^2+4x+4} dx = \int \frac{2x}{(x+2)^2} dx = \left[\frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{A(x+2)+B}{(x+2)^2} \Rightarrow \begin{matrix} A=2 \\ B=-4 \end{matrix}\right] =$$

$$= \int \frac{2}{x+2} - \frac{4}{(x+2)^2} dx = 2\ln|x+2| + \frac{4}{x+2} + C$$

c)

$$\int \frac{2x+1}{x^2+x-6} dx = \int \frac{2x+1}{(x+3)(x-2)} dx =$$

$$= \left[\frac{A}{x+3} + \frac{B}{x-2} = \frac{A(x-2)+B(x+3)}{(x+3)(x-2)} \Rightarrow \begin{matrix} A+B=2 \\ -2A+3B=1 \end{matrix} \Rightarrow \begin{matrix} A=1 \\ B=1 \end{matrix}\right] =$$

$$= \int \frac{1}{x+3} + \frac{1}{x-2} dx = \ln|x+3| + \ln|x-2| + C$$

d)

$$\int \frac{4x}{(1-2x)^2}dx = \left[\frac{A}{1-2x} + \frac{B}{(1-2x)^2} = \frac{A(1-2x)+B}{(1-2x)^2} \Rightarrow \begin{matrix} A=-2 \\ B=2 \end{matrix}\right] =$$

$$= 2\int \frac{1}{(1-2x)^2} - \frac{1}{1-2x}dx = \frac{1}{1-2x} + \ln|1-2x| + C$$

4317.

$$\int_2^5 \frac{x+3}{(x-1)^2}dx = \left[\frac{A}{(x-1)^2} + \frac{B}{x-1} = \frac{A+B(x-1)}{(x-1)^2} \Rightarrow \begin{matrix} A=4 \\ B=1 \end{matrix}\right] =$$

$$= \int_2^5 \frac{4}{(x-1)^2} + \frac{1}{x-1}dx = \left[\frac{4}{1-x} + \ln|x-1|\right]_2^5 =$$

$$\frac{4}{-4} + \ln 4 - \frac{4}{-1} - \ln 1 = 3 + \ln 4$$

4318.

$$\int_0^1 \frac{x+4}{(x+1)(x+2)}dx = \left\{\frac{A}{x+1} + \frac{B}{x+2} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)} \Rightarrow \begin{cases} A+B=1 \\ 2A+B=4 \end{cases} \Rightarrow \begin{cases} A=3 \\ B=-2 \end{cases}\right\} =$$

$$= \int_0^1 \frac{3}{x+1} - \frac{2}{x+2}dx = [3\ln|x+1| - 2\ln|x+2|]_0^1 =$$

$$= [3\ln 2 - 2\ln 3 - 3\ln 1 + 2\ln 2] = 5\ln 2 - 2\ln 3 = \ln\frac{32}{9}$$

4319.

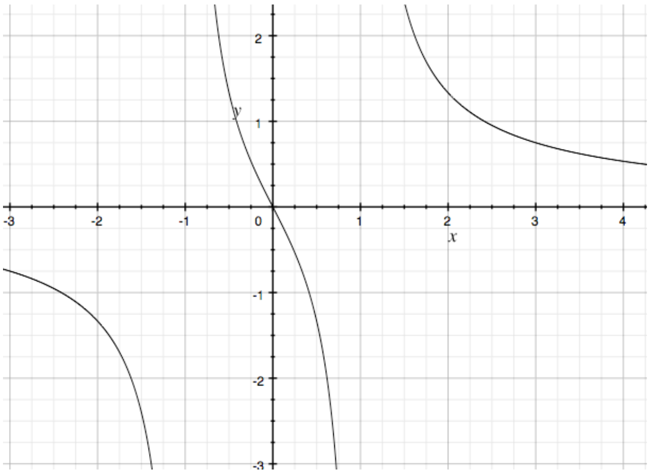
$$\int_1^e \frac{x+4}{x(x+1)}dx = \left\{\frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1)+Bx}{x(x+1)} \Rightarrow \begin{cases} A=4 \\ B=-3 \end{cases}\right\} =$$

$$= \int_1^e \frac{4}{x} - \frac{3}{x+1}dx = [4\ln|x| - 3\ln|x+1|\,]_1^e =$$

$$= [4\ln e - 3\ln(e+1) - 4\ln 1 + 3\ln 2] = 4 - 3\ln(e+1) + 3\ln 2 =$$

$$= 4 + 3\ln\frac{2}{e+1}$$

4320.



$$\int_2^4 \frac{2x}{x^2-1}dx = [\ln|x^2-1|]_2^4 = \ln 15 - \ln 3 = \ln 5 \text{ a. e.}$$

4321. a) $\int x \sin x \, dx = \begin{Bmatrix} f = x & f' = 1 \\ g' = \sin x & g = -\cos x \end{Bmatrix} = -x\cos x + \int \cos x \, dx = -x \cos x + \sin x + C$

b) $\int x \cos 2x \, dx = \begin{Bmatrix} f = x & f' = 1 \\ g' = \cos 2x & g = \frac{1}{2}\sin 2x \end{Bmatrix} = \frac{1}{2}x \sin 2x - \frac{1}{2}\int \sin 2x \, dx =$

$$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$$

c) $\int xe^{0.5x}dx = \begin{Bmatrix} f = x & f' = 1 \\ g' = e^{0.5x} & g = 2e^{0.5x} \end{Bmatrix} = 2xe^{0.5x} - 2\int e^{0.5x}dx = 2xe^{0.5x} - 4e^{0.5x} + C$

d) $\int (x+1)e^{2x}dx = \begin{Bmatrix} f = x+1 & f' = 1 \\ g' = e^{2x} & g = \frac{1}{2}e^{2x} \end{Bmatrix} = (x+1)\frac{1}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx =$

$$= (x+1)\frac{1}{2}e^{2x} - \frac{1}{4}e^{2x} + C = x\frac{1}{2}e^{2x} + \frac{1}{4}e^{2x} + C$$

4325.

$$\int \ln x \, dx = \int 1 \cdot \ln x \, dx = \begin{Bmatrix} f = \ln x & f' = \frac{1}{x} \\ g' = 1 & g = x \end{Bmatrix} = x \ln x - \int x\frac{1}{x}dx = x\ln x - x + C$$

4326. a) $\int x \ln x \, dx = \begin{Bmatrix} f = \ln x & f' = \frac{1}{x} \\ g' = x & g = \frac{1}{2}x^2 \end{Bmatrix} = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int \frac{1}{x}x^2 \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx =$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C = \frac{x^2}{4}(2\ln|x| - 1) + C$$

b) $\int \ln 2x \, dx = \int (\ln 2 + \ln x)dx = x \ln 2 + x \ln x - x + C = x \ln\ |2x| + C$

c) $\int \ln x^2 \, dx = 2\int \ln x \, dx = 2(x\ln|x| - x) + C$

4327.

$$\int_1^2 x\ln x\,dx = \{4326\ a)\} = \left[\frac{x^2}{4}(2\ln|x|-1)\right]_1^2 = \frac{2^2}{4}(2\ln|2|-1) - \left(\frac{1^2}{4}(2\ln|1|-1)\right) =$$

$$= \frac{4}{4}(2\ln 2-1) + \frac{1}{4} = 2\ln 2 - \frac{3}{4}$$

4328. a) $\int_1^e x^2\ln x\,dx = \begin{Bmatrix} f=\ln x & f'=\frac{1}{x} \\ g'=x^2 & g=\frac{1}{3}x^3 \end{Bmatrix} = \left[\frac{\ln x}{3}x^3\right]_1^e - \frac{1}{3}\int_1^e x^2 dx =$

$$= \left(\frac{\ln e}{3}e^3 - \frac{\ln 1}{3}1^3\right) - \frac{1}{9}[x^3]_1^e = \frac{e^3}{3} - \frac{1}{9}(e^3-1) = \frac{2}{9}e^3 + \frac{1}{9}$$

b) $\int_1^e x^2\ln x^2\,dx = \int_1^e x^2 2\ln x\,dx = 2\cdot\{\text{svaret i } 4328\ a)\} = \frac{2}{9}(2e^3+1)$

c) $\int_1^e x^2\ln\frac{1}{x}dx = -\int_1^e x^2\ln x\,dx = -1\cdot\{\text{svaret i } 4328\ a)\} = -\frac{1}{9}(2e^3+1)$

4329. a)

$$\int x^2\sin x\,dx = \begin{Bmatrix} f=x^2 & f'=2x \\ g'=\sin x & g=-\cos x \end{Bmatrix} = -x^2\cos x + 2\int x\cos x\,dx =$$

$$= \begin{Bmatrix} f=x & f'=1 \\ g'=\cos x & g=\sin x \end{Bmatrix} = -x^2\cos x + 2\left(x\sin x - \int \sin x\,dx\right) + C =$$

$$= -x^2\cos x + 2(x\sin x + \cos x) + C$$

b)

$$\int x^2e^{2x}dx = \begin{Bmatrix} f=x^2 & f'=2x \\ g'=e^{2x} & g=\frac{1}{2}e^{2x} \end{Bmatrix} = \frac{x^2}{2}e^{2x} - \frac{1}{2}\int 2xe^{2x}dx =$$

$$= \begin{Bmatrix} f=x & f'=1 \\ g'=e^{2x} & g=\frac{1}{2}e^{2x} \end{Bmatrix} = \frac{x^2}{2}e^{2x} - \left(\frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}dx\right) =$$

$$= \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C$$

c)

$$\int x^3e^x dx = \begin{Bmatrix} f=x^3 & f'=3x^2 \\ g'=e^x & g=e^x \end{Bmatrix} = x^3e^x - 3\int x^2e^x dx = \begin{Bmatrix} f=x^2 & f'=2x \\ g'=e^x & g=e^x \end{Bmatrix} =$$

$$= x^3e^x - 3\left(x^2e^x - 2\int xe^x dx\right) = x^3e^x - 3x^2e^x + 6\int xe^x dx =$$

$$= \begin{Bmatrix} f=x & f'=1 \\ g'=e^x & g=e^x \end{Bmatrix} = x^3e^x - 3x^2e^x + 6\left(xe^x - \int e^x dx\right) =$$

$$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$$

4330.

$$\int e^x\sin x\,dx = \begin{Bmatrix} f=e^x & f'=e^x \\ g'=\sin x & g=-\cos x \end{Bmatrix} = -e^x\cos x + \int e^x\cos x\,dx =$$

$$= \begin{Bmatrix} f=e^x & f'=e^x \\ g'=\cos x & g=\sin x \end{Bmatrix} = -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx \Rightarrow$$

$$\int e^x \sin x\,dx = e^x(\sin x - \cos x) - \int e^x \sin x\,dx \Rightarrow$$
$$\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C$$

4405.

$$y = x\sqrt{x} = x^{\frac{3}{2}} \Rightarrow y' = \frac{3}{2}\sqrt{x} \Rightarrow \int_0^5 \sqrt{1 + \frac{9}{4}x}dx = \frac{4}{9}\cdot\frac{2}{3}\left[\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}}\right]_0^5 =$$

$$= \frac{8}{27}\left(\left(1 + \frac{45}{4}\right)^{\frac{3}{2}} - 1\right) = \frac{8}{27}\left(\left(\frac{7}{2}\right)^3 - 1\right) = \frac{7^3 - 8}{27} = \frac{343 - 8}{27} = \frac{335}{27} \text{ l. e.}$$

4406.

$$y(x) = \frac{x^3}{6} + \frac{1}{2x} \Rightarrow y'(x) = \frac{x^2}{2} - \frac{1}{2x^2} = \frac{1}{2}\cdot\frac{x^4 - 1}{x^2} \Rightarrow y'^2(x) = \frac{1}{4x^4}(x^8 - 2x^4 + 1) =$$

$$= \frac{1}{4}\left(x^4 - 2 + \frac{1}{x^4}\right) \Rightarrow \int_1^2 \sqrt{1 + \frac{1}{4}\left(x^4 - 2 + \frac{1}{x^4}\right)}\,dx = \int_1^2 \sqrt{\frac{1}{4}\left(x^4 + 2 + \frac{1}{x^4}\right)}\,dx =$$

$$= \frac{1}{2}\int_1^2 \sqrt{\left(x^2 + \frac{1}{x^2}\right)^2}\,dx = \frac{1}{2}\int_1^2 x^2 + \frac{1}{x^2}\,dx = \frac{1}{2}\left[\frac{x^3}{3} - \frac{1}{x}\right]_1^2 = \frac{1}{2}\left(\frac{8}{3} - \frac{1}{2} - \left(\frac{1}{3} - 1\right)\right) =$$

$$= \frac{1}{2}\left(\frac{7}{3} + \frac{1}{2}\right) = \frac{17}{12} \text{ l. e.}$$

Test 4

1.

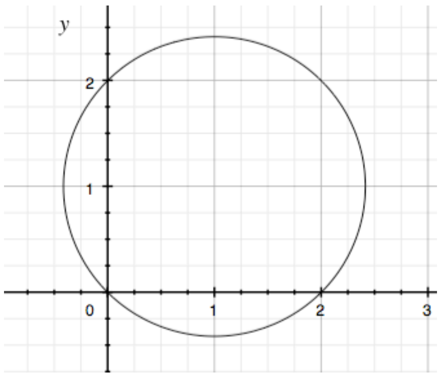
$$\int_0^{\infty} \frac{2}{e^x} dx = \lim_{w \to \infty} \int_0^{w} 2e^{-x} dx = 2 \lim_{w \to \infty} [e^{-x}]_w^0 = 2$$

2.

$$y = (x^2 + \sin x)^3 \Rightarrow y'(x) = 3(x^2 + \sin x)^2 (2x + \cos x)$$

3.

$$(x-1)^2 + (y-1)^2 = 2 \Rightarrow 2(1-x) + 2(y-1)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1-x}{y-1}, y'(0,0) = -1$$



4.

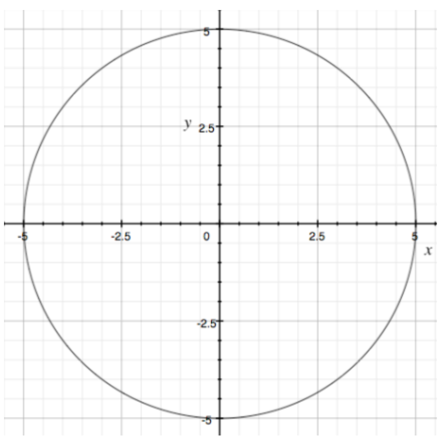
$$\int_0^{\infty} 2e^{-0.2} dx = \lim_{w \to \infty} \int_0^{w} 2e^{-0.2} dx = 2 \lim_{w \to \infty} [5e^{-0.2}]_w^0 = 10 \text{ a. e.}$$

5.

$$y^2 = 16x \Rightarrow 2y\frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y}$$

a) $y'(4) = \frac{8}{y} = 1$ b) $y = 4 + x$ c) $y = -4 - x$ (symmetriskäl)

6.



$$x^2+y^2=25, \frac{d}{dx} \Rightarrow 2x+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x}{y}, (4,\pm 3) \Rightarrow y'(4)=\pm\frac{4}{3}$$

7.

$$\frac{x}{x+4}=\frac{x+4-4}{x+4}=\frac{x+4}{x+4}+\frac{-4}{x+4}=1-\frac{4}{x+4}$$

8. a)

$$\int \frac{2x-3}{x}dx=\int \frac{2x}{x}-\frac{3}{x}dx \int 2-\frac{3}{x}dx=2x-3\ln|x|+C$$

b)

$$\int \frac{5x+2}{(x+1)(x-2)}dx=\int \frac{A}{x+1}+\frac{B}{x-2}dx=\int \frac{A(x-2)+B(x+1)}{(x+1)(x-2)}dx=$$

$$=\left\{\begin{matrix} A+B=5 \\ -2A+B=2 \end{matrix} \Rightarrow \begin{matrix} 3A=3 \\ B=4 \end{matrix}\right\}=\int \frac{1}{x+1}+\frac{4}{x-2}dx$$

9. a)

$$\int_0^1 2xe^x dx=\left\{\begin{matrix} f=x & f'=1 \\ g'=e^x & g=e^x \end{matrix}\right\}=2[xe^x]_0^1-2\int_0^1 e^x dx=2e-2[e^x]_0^1=2e-2(e-1)=2$$

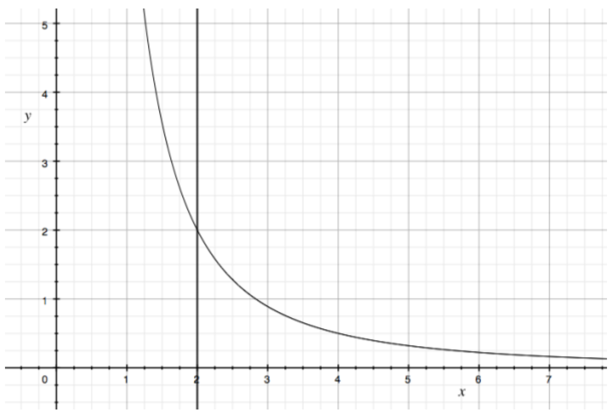
b)

$$\int_0^{\pi/4} 3x\sin 2x\,dx=\left\{\begin{matrix} f=x & f'=1 \\ g'=\sin 2x & g=-\frac{1}{2}\cos 2x \end{matrix}\right\}=\frac{3}{2}[x\cos 2x]_{\pi/4}^{0}+\frac{3}{2}\int_0^{\pi/4}\cos 2x\,dx=$$

$$=\frac{3}{2}(0-0)+\frac{3}{4}[\sin 2x]_0^{\pi/4}=\frac{3}{4}$$

10.

$$\int_2^\infty \frac{8}{x^2}dx=8\left[\frac{1}{x}\right]_\infty^2=4$$



11. a)

$$\int_0^1 -\frac{x}{2}e^{-x^2}dx = \frac{1}{4}\left[e^{-x^2}\right]_0^1 = \frac{e^{-1}-1}{4}$$

b)

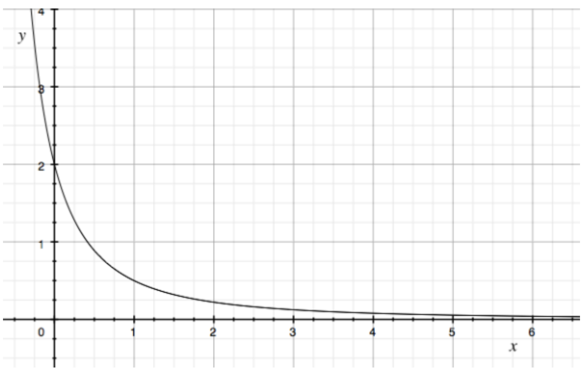
$$\int_0^\infty -\frac{x}{2}e^{-x^2}dx = \frac{1}{4}\left[e^{-x^2}\right]_0^\infty = -\frac{1}{4}$$

12.

$$\int_1^e x^3 \ln x\, dx = \begin{Bmatrix} f(x) = \ln x & f'(x) = \dfrac{1}{x} \\ g'(x) = x^3 & g(x) = \dfrac{x^4}{4} \end{Bmatrix} = \left[\frac{x^4}{4}\ln x\right]_1^e - \frac{1}{4}\int_1^e x^4 \frac{1}{x}dx =$$

$$= \frac{e^4}{4} - \frac{1}{4}\int_0^e x^3 dx = \frac{e^4}{4} - \frac{1}{16}[x^4]_1^e = \frac{e^4}{4} - \frac{1}{16}(e^4-1) = \frac{3e^4+1}{16}$$

13.



$$\int_0^\infty \frac{4\pi}{(x+1)^4}dx = \frac{4\pi}{3}\left[\frac{1}{(x+1)^3}\right]_\infty^0 = \frac{4\pi}{3}\ \text{v.e.}$$

14.

$$V = \frac{4\pi}{3}r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2 \Rightarrow dr = \frac{dV}{4\pi r^2} = \frac{2.4}{4\pi 1.6^2} \approx 0.075\ \text{dm/min}$$

Kapitel 4 Blandade uppgifter

3.

$$\lim_{n\to\infty}\int_1^n \frac{3dx}{x^2\sqrt{x}} = 3\lim_{n\to\infty}\int_1^n x^{-\frac{5}{2}}dx = 3\cdot\frac{2}{3}\lim_{n\to\infty}\left[x^{-\frac{3}{2}}\right]_n^1 = 2$$

4.

$$\frac{d}{dx}\left(2(x+2)^2+y^2\right) = \frac{d}{dx}(4) \Rightarrow 4(x+2)+2y\frac{dy}{dx} = 0 \Rightarrow$$

$$\frac{dy}{dx} = -\frac{2(x+2)}{y} \Rightarrow y'(-2) = 0$$

5.

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(2) \Rightarrow 2x+2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} \Rightarrow y'(1) = -1 \Rightarrow y = 2-x$$

6.

$$\frac{2x}{x+3} = \frac{2x+6-6}{x+3} = \frac{2x+6}{x+3} - \frac{6}{x+3} = 2 - \frac{6}{x+3}$$

7.

$$\frac{dr}{dt} = 4 \text{ och } A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r \Rightarrow \frac{dA}{dt} = \frac{dA}{dt}\frac{dr}{dr} = \frac{dA}{dr}\frac{dr}{dt} = 2\pi r\frac{dr}{dt} =$$

$$= 2\pi\cdot 20\cdot 4 = 160\pi \approx 500\ \text{cm}^2/\text{s} \ = 5\ \text{dm}^3/\text{s}$$

8. Konens volym är:

$$V = \frac{hb}{3} = \frac{h\pi r^2}{3} = \{\text{men } h = 3r\} = \frac{\pi h^3}{27} \Rightarrow \frac{dV}{dh} = \frac{\pi h^2}{9}$$

$$\frac{dh}{dt} = \frac{dh}{dt}\frac{dV}{dV} = \frac{dh}{dV}\frac{dV}{dt} = \left(\frac{dV}{dh}\right)^{-1}\frac{dV}{dt} = \frac{9}{\pi h^2}\frac{dV}{dt} = \frac{9}{\pi 3^2}1.5 \approx 4.8\ \text{cm/min}$$

9. a)

$$y = \left(e^{x^2}+x^2\right)^5 \Rightarrow y' = 5\left(e^{x^2}+x^2\right)^4\left(2xe^{x^2}+2x\right) = 10x\left(e^{x^2}+x^2\right)^4\left(e^{x^2}+1\right)$$

b)

$$y = \frac{5}{(\cos x - x^3)^2} \Rightarrow y' = \frac{5(-2)(-\sin x - 3x^2)}{(\cos x - x^3)^3} = \frac{10(\sin x + 3x^2)}{(\cos x - x^3)^3}$$

10. a)

$$x^2+y+y^2 = 1 \Rightarrow 2x+\frac{dy}{dx}+2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{1+2y}$$

b)

$$x^2 \cdot y^2 - 4 = 0, \frac{d}{dx} \Rightarrow 2x \cdot y^2 + x^2 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

11.

$$V = \int_{-1}^{\infty} \pi e^{-2(x+1)} dx = \pi \left[\frac{1}{2} e^{-2(x+1)} \right]_{\infty}^{-1} = \frac{\pi}{2} \text{ v.e.}$$

12. a)

$$\frac{5x}{(x-1)(x+4)} = \frac{A}{x-1} + \frac{B}{x+4} = \frac{A(x+4) + B(x-1)}{(x-1)(x+4)} =$$

$$= \begin{cases} A + B = 5 \\ 4A - B = 0 \end{cases} \Rightarrow \begin{Bmatrix} A = 1 \\ B = 4 \end{Bmatrix} = \frac{1}{x-1} + \frac{4}{x+4}$$

b)

$$\frac{5}{(x-1)(x+4)} = \frac{A}{x-1} + \frac{B}{x+4} = \frac{A(x+4) + B(x-1)}{(x-1)(x+4)} =$$

$$= \begin{cases} A + B = 0 \\ 4A - B = 5 \end{cases} \Rightarrow \begin{Bmatrix} A = 1 \\ B = -1 \end{Bmatrix} = \frac{1}{x-1} - \frac{1}{x+4}$$

13. a)

$$\int \frac{x}{x+4} dx = \int \frac{x+4-4}{x+4} dx = \int \frac{x+4}{x+4} - \frac{4}{x+4} dx =$$
$$= \int 1 - \frac{4}{x+4} dx = x - 4\ln|x+4| + C$$

b)

$$\int \frac{x+2}{x+4} dx = \int \frac{x+4-2}{x+4} dx = \int \frac{x+4}{x+4} - \frac{2}{x+4} dx =$$
$$= \int 1 - \frac{2}{x+4} dx = x - 2\ln|x+4| + C$$

14. a)

$$\int_0^{0.5} 2xe^{2x} dx = \begin{Bmatrix} f = x & f' = 1 \\ g' = e^{2x} & g = \frac{1}{2} e^{2x} \end{Bmatrix} = 2\left[x \frac{1}{2} e^{2x} \right]_0^{0.5} - 2 \int_0^{0.5} \frac{1}{2} e^{2x} dx =$$
$$= \frac{e}{2} - \frac{1}{2} [e^{2x}]_0^{0.5} = \frac{e}{2} - \frac{e}{2} + \frac{1}{2} = \frac{1}{2}$$

b)

$$\int_0^4 \frac{2x}{\sqrt{x^2+9}} dx = \left[2\sqrt{x^2+9} \right]_0^4 = 2\left(\sqrt{4^2+9} - \sqrt{0^2+9} \right) =$$
$$= 2(5-3) = 4$$

15.

$$V=\int_{-\infty}^{\infty}\pi e^{-2|x|}dx=2\pi\lim_{w\to\infty}\int_{0}^{w}e^{-2x}dx=2\pi\lim_{w\to\infty}\left[\frac{1}{2}e^{-2x}\right]_{w}^{0}=\pi\text{ v.e.}$$

16.

$$y=\frac{1}{2}\ln x-\frac{x^2}{4}\Rightarrow y'=\frac{1}{2x}-\frac{x}{2}\Rightarrow (y')^2=\frac{1}{4x^2}-\frac{1}{2}+\frac{x^2}{4}$$

$$\int_1^e\sqrt{1+(y')^2}dx=\int_1^e\sqrt{\frac{1}{4x^2}+\frac{1}{2}+\frac{x^2}{4}}dx=\int_1^e\sqrt{\left(\frac{1}{2x}+\frac{x}{2}\right)^2}dx=$$

$$=\frac{1}{2}\int_1^e\left(\frac{1}{x}+x\right)dx=\frac{1}{2}\left[\ln x+\frac{x^2}{2}\right]_1^e=\frac{1+e^2}{4}$$

17.

$$42\frac{\text{cm}^2}{\text{min}}=\frac{dA}{dt}=\frac{dA}{dt}\frac{dr}{dr}=\frac{dA}{dr}\frac{dr}{dt}=\frac{d}{dr}(4\pi r^2)\frac{dr}{dt}=8\pi r\frac{dr}{dt}=42\text{ cm}^2/\text{min}$$

$$\frac{dV}{dt}=\frac{dV}{dt}\frac{dr}{dr}=\frac{dV}{dr}\frac{dr}{dt}=\frac{d}{dr}\left(\frac{4\pi r^3}{3}\right)\frac{dr}{dt}=4\pi r^2\frac{dr}{dt}$$

Men då:

$$\frac{dr}{dt}=\frac{42}{8\pi r}\Rightarrow\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}=4\pi r^2\frac{42}{8\pi r}=21r=252\text{ cm}^3/\text{min}$$

18.

$$y=\frac{x^3}{12}+\frac{1}{x}\Rightarrow y'=\frac{x^2}{4}-\frac{1}{x^2}\Rightarrow (y')^2=\frac{x^4}{16}-\frac{1}{2}+\frac{1}{x^4}$$

$$S=\int_1^4\sqrt{1+\frac{x^4}{16}-\frac{1}{2}+\frac{1}{x^4}}dx=\int_1^4\sqrt{\frac{x^4}{16}+\frac{1}{2}+\frac{1}{x^4}}dx=$$

$$=\int_1^4\sqrt{\left(\frac{x^2}{4}+\frac{1}{x^2}\right)^2}dx=\int_1^4\frac{x^2}{4}+\frac{1}{x^2}dx=\left[\frac{x^3}{12}-\frac{1}{x}\right]_1^4=\frac{4^3}{12}-\frac{1}{4}-\left(\frac{1}{12}-\frac{1}{1}\right)=$$

$$=\frac{64}{12}-\frac{3}{12}-\frac{1}{12}+\frac{12}{12}=\frac{72}{12}=6\text{ l.e.}$$

Omfångsrika uppgifter

(sid 190 ff)

1.

A: $\int_0^1 e^{-x}dx=[e^{-x}]_1^0=1-e^{-1}\approx 0.63$

B: $\int_0^1\frac{e^x}{1+e^x}dx=[\ln(1+e^x)]_0^1=\ln(1+e)-\ln(1+1)=\ln\frac{1+e}{2}\approx 0.62$

C: $\int_{-1}^0 e^x\cos(e^x)dx=[\sin(e^x)]_{-1}^0=\sin(1)-\sin(e^{-1})\approx 0.48$

2. $x^2 + y^2 = 25, \frac{d}{dx} \Rightarrow 2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} = -\frac{4}{3}$ i (4,3)

För att hitta var tangenten skär x-axeln kan m hittas:

$$y = -\frac{4}{3}x + m \Rightarrow 3 = -\frac{4}{3}4 + m \Rightarrow m = 3 + \frac{16}{3} \Rightarrow y = -\frac{4}{3}x + \frac{25}{3}$$

$$y = 0 \Rightarrow x = \frac{25}{4} = 6.25 \Rightarrow A = A_{\triangle} - A_{\measuredangle} = \frac{6.25 \cdot 3}{2} - \pi 5^2 \frac{1}{2\pi} \arctan\left(\frac{3}{4}\right) \approx 1.33 \text{ a. e.}$$

Eller, kanske enklare, är att inse att de båda trianglarna är kongruenta 3, 4, 5-trianglar. Detta gör att den stora triangelns 3-sida sammanfaller med den lilla triangelns 4 sida. Detta ger samma svar som ovan. Och man slipper den implicita deriveringen!

18.

$$\frac{dy}{y(N-y)} = kdt \Rightarrow dy\left(\frac{A}{y} + \frac{B}{N-y}\right) = kdt \Rightarrow dy\frac{A(N-y) + By}{y(N-y)} = kdt = \left\{\begin{matrix} A = \frac{1}{N} \\ B = \frac{1}{N} \end{matrix}\right\} \Rightarrow$$

$$\frac{1}{N}dy\left(\frac{1}{y} + \frac{1}{N-y}\right) = kdt \Rightarrow \{\text{integrera båda sidor}\} \Rightarrow \frac{1}{N}\big(\ln y - \ln(N-y)\big) = kt + C$$

$$\ln\frac{y}{N-y} = N(kt + C) \Rightarrow \frac{y}{N-y} = e^{N(kt+C)} \Rightarrow y = Ne^{N(kt+C)} - ye^{N(kt+C)} \Rightarrow$$

$$y(t) = \frac{Ne^{N(kt+C)}}{1 + e^{N(kt+C)}} = \frac{Ne^{Nkt}e^{NC}}{1 + e^{Nkt}e^{NC}} = \frac{Ne^{Nkt}}{C_1 + e^{Nkt}}$$

19. a) $x^3 + y^3 = 3xy,\ \frac{d}{dx}(x^3 + y^3 = 3xy) \Rightarrow 3x^2 + 3y^2\frac{dy}{dx} = 3y + 3x\frac{dy}{dy}$

$$\frac{dy}{dx}(x - y^2) = x^2 - y \Rightarrow \frac{dy}{dx} = \frac{x^2 - y}{x - y^2}$$

$$\frac{dy}{dx} = 0 \text{ då } x^2 = y \Rightarrow x^3 + x^6 = 3x^3 \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \text{och} \begin{cases} x = \sqrt[3]{2} \\ y = \sqrt[3]{4} \end{cases}$$

b) $\frac{dy}{dx} = \infty$ då $x = y^2$ dvs $y^6 + y^3 = 3y^3 \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$ och $\begin{cases} x = \sqrt[3]{4} \\ y = \sqrt[3]{2} \end{cases}$

c) $x^3 + y^3 = 3xy \Rightarrow x^3 + t^3x^3 = 3xtx \Rightarrow x = \frac{3t}{1+t^3}, y = \frac{3t^2}{1+t^3}, t \neq -1$

20. Produkten av avstånden till $(-a, 0)$ och $(a, 0)$ fås som:

$$\sqrt{\big(x - (-a)\big)^2 + y^2} \cdot \sqrt{(x-a)^2 + y^2} = a^2 \Rightarrow$$

$$((x+a)^2 + y^2)((x-a)^2 + y^2) = a^4 \Rightarrow$$

$$(x^2 + 2xa + a^2 + y^2)(x^2 - 2xa + a^2 + y^2) = a^4 \Rightarrow$$

$$x^4 - x^3 2a + x^2a^2 + x^2y^2 +$$
$$2ax^3 - 4x^2a^2 + 2xa^3 + 2xay^2 +$$
$$a^2x^2 - 2xa^3 + a^4 + a^2y^2 +$$
$$y^2x^2 - 2xay^2 + a^2y^2 + y^4 = a^4$$

$$x^4 + 2x^2y^2 + y^4 = 2x^2a^2 - 2a^2y^2 \Rightarrow$$

$$(x^2+y^2)^2 = 2a^2(x^2 - y^2) \text{ VSV}$$

b)

$$\frac{d}{dx}(x^2+y^2)^2 = \frac{d}{dx}2a^2(x^2 - y^2)$$

$$2(x^2+y^2)\left(2x + 2y\frac{dy}{dx}\right) = 2a^2\left(2x - 2y\frac{dy}{dx}\right)$$

$$x(x^2+y^2) + y(x^2+y^2)\frac{dy}{dx} = a^2x - a^2y\frac{dy}{dx}$$

$$y(x^2+y^2)\frac{dy}{dx} + a^2y\frac{dy}{dx} = a^2x - x(x^2+y^2)$$

$$\frac{dy}{dx}\left(a^2y + y(x^2+y^2)\right) = a^2x - x(x^2+y^2)$$

$$\frac{dy}{dx} = 0 \text{ då } a^2x = x(x^2+y^2) \Rightarrow x^2+y^2 = a^2$$

Uttrycket är en cirkel med mittpunkt i origo och radie *a*, dvs 4.

$$x^2 = 4^2 - 2^2 \Rightarrow \begin{cases} x = \pm 2\sqrt{3} \\ y = \pm 2 \end{cases}$$

21. Personen befinner sig på avståndet $\sqrt{20}$ km från fyren. Käglan roterar ett varv på 20 s. Käglans hastighet torde bli:

$$v = \frac{2\pi\sqrt{20}}{20} \approx 1.4 \text{ km/s}$$

(Facit ger 3.1 km/s, oklart varför.)

22. Om papprets kortsida sätts till 1 l.e. blir långsidan $\sqrt{2}$ l.e.. Kallas den skuggade triangelns vertikala katet $1 - y$ fås:

$$(1-y)^2 + x^2 = y^2 \Leftrightarrow 1 - 2y + y^2 + x^2 = y^2 \Leftrightarrow y = \frac{1}{2}(x^2 + 1)$$

Triangelns area kan uttryckas:

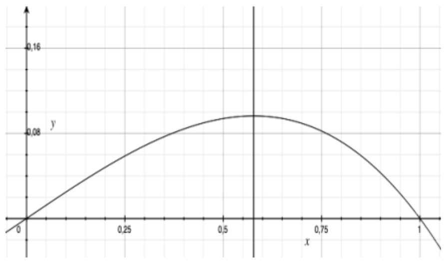
$$A(x) = \frac{1}{2}x(1-y) = \frac{1}{2}x\left(1 - \frac{1}{2}(x^2+1)\right) = \frac{1}{2}x\left(\frac{1}{2} - \frac{x^2}{2}\right) = \frac{1}{4}(x - x^3)$$

Med hjälp av derivatan kan den största arean sökas:

$$\frac{dA(x)}{dx} = \frac{1}{4}(1 - 3x^2) = 0 \text{ då } x = \frac{1}{\sqrt{3}} \approx 0.58$$

Dvs

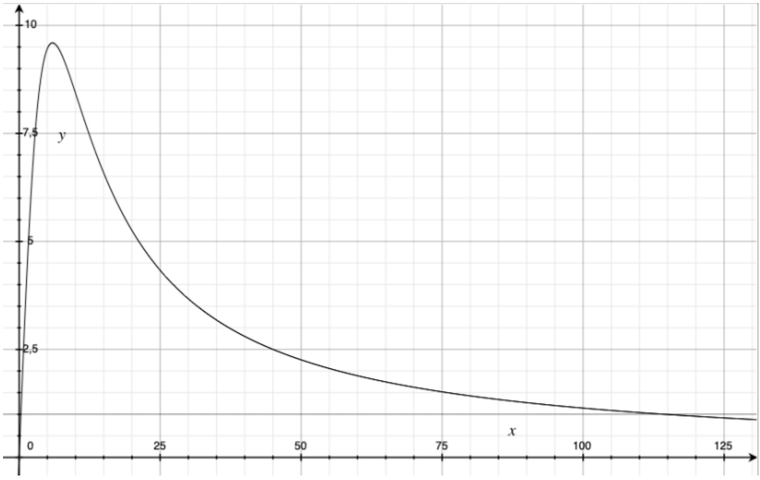
$$A_{max} = \frac{1}{4}x(1-x^2) = \frac{1}{4}\frac{1}{\sqrt{3}}\left(1 - \frac{1}{3}\right) = \frac{1}{6\sqrt{3}} \approx 0.096 \text{ a. e.}$$



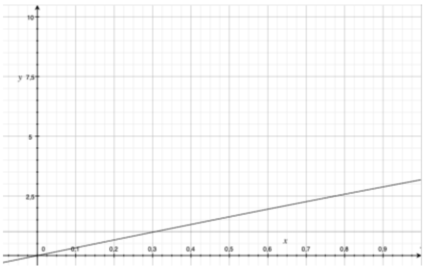
23. Funktionen $v(d)$ visar vinkelns beroende av avståndet d.

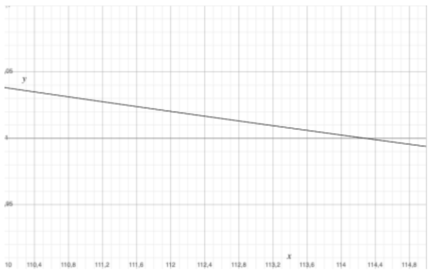
a)

$$v(d) = \arctan\frac{7}{d} - \arctan\frac{5}{d}$$



b) Detaljstudier gällande intervallen $0 < d < 1$ m och $110 < d < 115$ m nedan:





Tillsammans ger dessa intervallet $0.3 < d < 114$ m.

c) Cirka 5-6 meter.

d)

$$v'(d) = -\frac{7}{d^2}\frac{1}{1+\left(\frac{7}{d}\right)^2} + \frac{5}{d^2}\frac{1}{1+\left(\frac{5}{d}\right)^2} = 0$$

$$\frac{7}{d^2}\frac{1}{1+\left(\frac{7}{d}\right)^2} = \frac{5}{d^2}\frac{1}{1+\left(\frac{5}{d}\right)^2} \Rightarrow 7\left(1+\left(\frac{5}{d}\right)^2\right) = 5\left(1+\left(\frac{7}{d}\right)^2\right) \Rightarrow$$

$$7\left(1+\frac{25}{d^2}\right) = 5\left(1+\frac{49}{d^2}\right) \Rightarrow 7+\frac{175}{d^2} = 5+\frac{245}{d^2} \Rightarrow 2 = \frac{70}{d^2} \Rightarrow d = \sqrt{35} \approx 5.9 \text{ m}$$

24.

$$\frac{\int_{-2}^{0} x^3 - x^2 - 4x + 3 - (-x^2 + 3)dx}{\int_0^2 -x^2 + 3 - (x^3 - x^2 - 4x + 3)\,dx} = \frac{\int_{-2}^{0} x^3 - 4xdx}{-\int_0^2 x^3 - 4x\,dx} = \frac{\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0}}{\left[\frac{x^4}{4} - 2x^2\right]_{2}^{0}} =$$

$$= \frac{\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0}}{\left[\frac{x^4}{4} - 2x^2\right]_{2}^{0}} = \frac{4}{4} = 1$$

27. a)

$$\frac{d}{dx}\cosh x = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x + e^{-x}}{2} = \sinh x$$

$$\frac{d}{dx}\sinh x = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) = \frac{e^x - e^{-x}}{2} = \cosh x$$

$$\frac{d}{dx}\tanh x = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right) = \frac{e^x - e^{-x}}{e^x - e^{-x}} - \frac{e^x + e^{-x}}{(e^x - e^{-x})^2}(e^x + e^{-x}) =$$

$$= \frac{e^{2x} - 2 + e^{-2x}}{(e^x - e^{-x})^2} - \frac{e^{2x} + 2 + e^{-2x}}{(e^x - e^{-x})^2} = -\frac{2^2}{(e^x - e^{-x})^2} = -\frac{1}{\sinh^2 x}$$

En intressant integral, Gaussklockan:

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx$$

Låt oss se om vi kan hitta I^2:

$$I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2} dx \right)^2 = \int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-y^2} dy \, e^{-x^2} dx =$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(y^2+x^2)} dy \, dx = \left\{ \begin{matrix} dydx = \rho d\varphi d\rho \\ y^2 + x^2 = \rho^2 \\ 0 \leq \varphi \leq 2\pi \\ 0 \leq \rho \leq \infty \end{matrix} \right\} = \int_0^{\infty} \int_0^{2\pi} e^{-\rho^2} \rho d\rho \, d\varphi =$$

$$= 2\pi \frac{1}{2} \left[e^{-\rho^2} \right]_{\infty}^{0} = \pi \Rightarrow I = \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Talkodning:

$$s_w(n) = s(n) + \sum_{i=1}^{10} a_i \gamma_1^i s(n-i) - \sum_{i=1}^{10} a_i \gamma_2^i s_w(n-i) \,, \quad n = 0 \cdots L-1$$

$$s_w(n) = s(n) + a_1 \gamma_1 s(n-1) + \cdots + a_{10} \gamma_1^{10} s(n-10)$$

$$-\left(a_1 \gamma_2 s_w(n-1) + \cdots + a_{10} \gamma_2^{10} s_w(n-10) \right)$$

där , $n = 0 \cdots 159$

eller:

$$s_w(n) = s(n) + a_1 0.9_1 s(n-1) + \cdots + a_{10} 0.9^{10} s(n-10)$$

$$-\left(a_1 0.6 s_w(n-1) + \cdots + a_{10} 0.6^{10} s_w(n-10) \right)$$

$$D = \sum_{n=1}^{40} \big(x(n) - gx(n-L)\big)^2 = \sum x(n)^2 - 2g \sum x(n)x(n-L) + \sum g^2 x^2(n-L)$$

$$\frac{\partial D}{\partial g} = -2 \sum x(n)x(n-L) + 2g \sum x^2(n-L) = 0$$

$$g = \frac{\sum x(n)x(n-L)}{\sum x^2(n-L)}$$

$$D = \sum x^2(n) - 2g \sum x(n)x(n-L) + \sum g^2 x^2(n-L) =$$

$$= \sum x^2(n) - 2\frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} + \left[\frac{\sum x(n)x(n-L)}{\sum x^2(n-L)}\right]^2 \sum x^2(n-L) =$$

$$= \sum x^2(n) - 2\frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} + \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)} =$$

$$= \sum x^2(n) - \frac{[\sum x(n)x(n-L)]^2}{\sum x^2(n-L)}$$

Testa $41 \leq L \leq 169$