

Valda uppgifter i kursboken Matematik M2c av Sjunnesson med flera utgiven på Liber, (2011).

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Kapitel 1

$$1113. \text{ a) } 1.4x - \frac{2}{5} \cdot (5 + 3.5x) = 1.4x - \left(2 + \frac{7}{5}x\right) = 1.4x - 2 - 1.4x = -2$$

$$\text{b) } 1.5(2x - y) - (x - 0.6y) \cdot 3 = 3x - 1.5y - (3x - 1.8y) = 0.3y$$

$$1114. \text{ a) } \frac{2}{3}(9 - 3x + 2y) - \left(\frac{y}{3} - \frac{x}{4} + 0.5\right) \cdot 4 = 6 - 2x + \frac{4}{3}y - \left(\frac{4}{3}y - x + 2\right) = 4 - x$$

$$\text{b) } 4 - x = 4 - (-5) = 4 + 5 = 9$$

$$1115. \text{ a) } C = A - B = 3 - 5x - (x + 3) = 3 - 5x - x - 3 = -6x$$

$$\text{b) } C = 2B - 3A = 2(x + 3) - 3(3 - 5x) = 2x + 6 - 9 + 15x = 17x - 3$$

$$\text{c) } C = 3B - Ax = 3(x + 3) - (3 - 5x)x = 3x + 9 - 3x + 5x^2 = 9 + 5x^2$$

$$1116. \text{ a) } \frac{x}{2} - \frac{3x}{2} + \frac{5x}{2} - \frac{7x}{2} + \frac{9x}{2} = \frac{x-3x+5x-7x+9x}{2} = \frac{5x}{2}$$

$$\text{b) } \frac{5x}{2} = \frac{5 \cdot 16}{2} = 40 \quad \text{c) } \frac{5x}{2} = \frac{5}{2} \cdot \left(-\frac{2}{15}\right) = -\frac{1}{3}$$

$$1117. \frac{7-2x}{3} - \frac{5-2x}{3} = \frac{7-2x-(5-2x)}{3} = \frac{7-2x-5+2x}{3} = \frac{2}{3}$$

$$1118. \text{ a) } \bar{x} = \frac{1}{4}(2x + 9 + 13 + x + 3(x - 8) + 2(1 + x)) = \frac{1}{4}8x = 2x$$

$$\text{b) } \bar{x} = 2 \cdot 9.5 = 19 \quad \text{c) } \bar{x} = -2 \cdot \frac{3}{8} = -\frac{3}{4}$$

$$1119. \text{ a) } \sqrt{3}(\sqrt{12} + \sqrt{3}) = (\sqrt{3}\sqrt{12} + \sqrt{3}\sqrt{3}) = \sqrt{3 \cdot 12} + 3 = \sqrt{36} + 3 = 9$$

$$\text{b) } \sqrt{2}(\sqrt{8} - \sqrt{18}) = (\sqrt{2}\sqrt{8} - \sqrt{2}\sqrt{18}) = \sqrt{16} - \sqrt{36} = 4 - 6 = -2$$

$$1120. 1 + x(2 - x) \cdot x - x^2(1 - x) = 1 + x^2(2 - x) - x^2(1 - x) = \\ = 1 + (2x^2 - x^3) - (x^2 - x^3) = 1 + x^2 \geq 1 > 0$$

1135.

$$2(x + 4) + x = 50 \Rightarrow 3x = 42 \Rightarrow x = 14 \text{ cm}$$

1136.

$$(400 + 4x) \cdot 3 + 2(350 + 3x) = 2800 \Rightarrow$$

$$1200 + 12x + 700 + 6x = 2800 \Rightarrow 18x = 900 \Rightarrow x = 50$$

1137. a)

$$\begin{cases} B = 5000 + E \\ F = 3 \cdot E \\ B + F + E = 20\,000 \end{cases} \Rightarrow 5000 + E + 3E + E = 20\,000 \Rightarrow \begin{cases} E = 3000 \\ B = 8000 \\ F = 9000 \end{cases}$$

$$\text{b) } \begin{cases} B = 5000 + E \\ F = 3 \cdot E \\ B + F + E = 40\,000 \end{cases} \Rightarrow 5000 + E + 3E + E = 40\,000 \Rightarrow \begin{cases} E = 7\,000 \\ B = 12\,000 \\ F = 21\,000 \end{cases}$$

$$1138. \text{ a) } 20 + \frac{x}{6} = x \Rightarrow 120 + x = 6x \Rightarrow x = \frac{120}{5} = 24$$

$$\text{b) } \frac{y}{3} + \frac{y}{2} = 15 \Rightarrow 6\frac{y}{3} + 6\frac{y}{2} = 6 \cdot 15 \Rightarrow 2y + 3y = 90 \Rightarrow y = \frac{90}{5} = 18$$

$$1139. \text{ a) } \frac{3p}{2} + \frac{9}{10} = \frac{7p}{5} \{\text{förläng med } 10\} \Rightarrow 5 \cdot 3p + 9 = 2 \cdot 7p \Rightarrow p = -9$$

$$\text{b) } \frac{1}{x} + \frac{2}{5} = \frac{3}{2x} \{\text{förläng med } 10x\} \Rightarrow 10 + 4x = 15 \Rightarrow x = \frac{5}{4}$$

1140. a)

$$9 - \frac{3(4x - 2)}{5} = \frac{2 + x}{10}$$

förläng med 10:

$$90 - 6(4x - 2) = 2 + x \Rightarrow 90 - 24x + 12 = 2 + x \Rightarrow 100 = 25x \Rightarrow x = 4$$

b)

$$\frac{6y - 2}{5} - \frac{4y - 2}{3} = 1 - \frac{8 + y}{6}$$

förläng med 30:

$$30 \cdot \frac{6y - 2}{5} - 30 \cdot \frac{4y - 2}{3} = 30 \cdot \left(1 - \frac{8 + y}{6}\right)$$

$$6 \cdot (6y - 2) - 10 \cdot (4y - 2) = 30 - 5(8 + y)$$

$$36y - 12 - (40y - 20) = 30 - (40 + 5y)$$

$$36y - 12 - 40y + 20 = 30 - 40 - 5y$$

$$8 - 4y = -10 - 5y \Rightarrow y = -18$$

1141.

$$x - 12 = \frac{x}{3} \Rightarrow 3x - 36 = x \Rightarrow 2x = 36 \Rightarrow x = 18$$

$$1142. \begin{cases} J = 2 \cdot E \\ M = J + 4 \\ E + J + M = 64 \end{cases} \Rightarrow \begin{cases} J = 2 \cdot E \\ M = 2E + 4 \\ E + 2E + 2E + 4 = 64 \end{cases} \Rightarrow 5E = 60 \Rightarrow M = 28 \text{ år}$$

$$1143. \frac{x+2}{3x+6} = \frac{1}{3} \Rightarrow \frac{x+2}{3(x+2)} = \frac{1}{3} \text{ gäller för alla } x \text{ utom } x = -2.$$

$$1144. \text{ a) } \frac{1}{6} = \frac{2}{x-1} \Rightarrow x - 1 = 12 \Rightarrow x = 13$$

$$\text{ b) } \frac{3}{x-2} = \frac{2}{x+1} \Rightarrow 3(x+1) = 2(x-2) \Rightarrow x = -7$$

$$1145. \frac{x}{25+x} = 0.02 \Rightarrow x = 0.5 + 0.02x \Rightarrow 0.98x = 0.5 \Rightarrow x = \frac{0.5}{0.98} \approx 5.1 \text{ dl}$$

$$1146. \frac{1}{6} = \frac{2}{3-x^2} \Rightarrow 3 - x^2 = 12 \Rightarrow x^2 = -9 \text{ dvs inga reella rötter.}$$

1211.

$$\begin{aligned} 3x(2x-5) - (x+8)(x-1) + 2(2.5x-1)(4-x) &= \\ = 6x^2 - 15x - (x^2 - x + 8x - 8) + (5x-2)(4-x) &= \\ = 6x^2 - 15x - x^2 + x - 8x + 8 + 20x - 5x^2 - 8 + 2x &= \\ = -15x + x - 8x + 20x + 2x &= 0 \end{aligned}$$

1213.

$$\begin{aligned} 3 - (x-7)(2x+6) &= 3 - (2x^2 + 6x - 14x - 42) = \\ &= 3 - 2x^2 - 6x + 14x + 42 = 45 + 8x - 2x^2 \end{aligned}$$

1214.

$$\begin{aligned} (x+y+2)(3+x) - (x-y)(-3-x) &= (x+y+2)(3+x) + (x-y)(3+x) = \\ &= (x+y+2+(x-y))(3+x) = (2+2x)(3+x) = 6 + 8x + 2x^2 \end{aligned}$$

1215. Kalla sidorna x , y , u och z analogt med problem 1212. Då gäller:

$$\begin{cases} xy = 117 \\ uy = 135 \\ xz = 156 \end{cases} \Rightarrow uz = \frac{135}{y} \cdot \frac{156}{x} = \frac{135 \cdot 156}{117} = 180 \text{ m}^2$$

$$1225. (a-b) \cdot (a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$$

$$1226. (a-b)^2 = a^2 - 2ab + b^2 = b^2 - 2ab + a^2 = (b-a)^2$$

$$1227. \text{ a) } (\sqrt{2} + \sqrt{8})^2 = (\sqrt{2})^2 + 2\sqrt{2}\sqrt{8} + (\sqrt{8})^2 = 2 + 2\sqrt{16} + 8 = 18$$

$$\text{ b) } (\sqrt{12} - \sqrt{3})^2 = (\sqrt{12})^2 - 2\sqrt{12}\sqrt{3} + (\sqrt{3})^2 = 12 - 2\sqrt{36} + 3 = 3$$

1228. a)

$$(x+h)^3 - (x-h)^3 = x^3 + 3x^2h + 3xh^2 + h^3 - (x^3 - 3x^2h + 3xh^2 - h^3) =$$

$$= x^3 + 3x^2h + 3xh^2 + h^3 - x^3 + 3x^2h - 3xh^2 + h^3 = 6x^2h + 2h^3$$

b)

$$(x+2)^3 - (x+2)^2 - (x-2) =$$

$$= x^3 + 3x^2 \cdot 2 + 3x \cdot 2^2 + 2^3 - (x^2 + 4x + 4) - (x-2) =$$

$$= x^3 + 6x^2 + 12x + 8 - x^2 - 4x - 4 - x + 2 =$$

$$= x^3 + 5x^2 + 7x + 6$$

$$1229 \text{ a) } (\sqrt{2x} + \sqrt{4.5x})^2 = 2x + 2\sqrt{2x}\sqrt{4.5x} + 4.5x = 6.5x + 2 \cdot 3x = 12.5x$$

$$\text{b) } (\sqrt{20x} - x\sqrt{5x})^2 = 20x - 2\sqrt{20x} \cdot x\sqrt{5x} + x^2 5x = 20x - 2x \cdot 10x + x^2 5x =$$

$$= 20x - 20x^2 + 5x^3$$

$$\text{c) } (5^{3x} - 5^{0.5x})^2 = 5^{6x} - 2 \cdot 5^{3x} 5^{0.5x} + 5^x = 5^{6x} - 2 \cdot 5^{3.5x} + 5^x$$

$$\text{d) } \left(\frac{a+1}{2}\right)^2 - \left(\frac{a-1}{2}\right)^2 = \frac{a^2+2a+1}{4} - \frac{a^2-2a+1}{4} = \frac{a^2+2a+1-a^2+2a-1}{4} = a$$

1230. Hela den stora kvadraten = 4 gula plus den röda. Eller matematiskt uttryckt:

$$c^2 = 4 \frac{ab}{2} + (a-b)^2 = 2ab + a^2 - 2ab + b^2 = a^2 + b^2$$

1243.

$$\begin{cases} a-b=5 \\ a^2-b^2=195 \end{cases} \Rightarrow \begin{cases} a-b=5 \\ (a+b)(a-b)=195 \end{cases} \Rightarrow \begin{cases} a-b=5 \\ (a+b)5=195 \end{cases} \Rightarrow \begin{cases} a-b=5 \\ a+b=39 \end{cases}$$

$$\Rightarrow \begin{cases} a=22 \\ b=17 \end{cases}$$

$$1244. \text{ a) } 18^2 = 20 \cdot 16 + 4 = 320 + 4 = 324, \quad 14^2 = 10 \cdot 18 + 16 = 196$$

$$39^2 = 40 \cdot 38 + 1 = 1520 + 1 = 1521, \quad 97^2 = 100 \cdot 94 + 9 = 9409$$

$$\text{b) } a^2 = (a-b)(a+b) + b^2 = a^2 - b^2 + b^2 = a^2 \text{ VSV}$$

1254.

$$(x+3)^2 + 8(3-x)^2 = 43 - (5+3x)(5-3x)$$

$$(x^2 + 6x + 9) + 8(9 - 6x + x^2) = 43 - (25 - 9x^2)$$

$$x^2 + 6x + 9 + 72 - 48x + 8x^2 = 43 - 25 + 9x^2$$

$$9x^2 + 81 - 42x = 18 + 9x^2 \Rightarrow 63 = 42x \Rightarrow x = \frac{63}{42} = \frac{3}{2} = 1.5$$

$$1255. (3x + 4)^2 - (4 - 3x)^2 = (2x + 3)^2 - (3 - 2x)^2$$

$$(9x^2 + 24x + 16) - (16 - 24x + 9x^2) = (4x^2 + 12x + 9) - (4x^2 - 12x + 9)$$

$$48x = 24x \Rightarrow x = 0$$

$$1256. x^2 + x + 19 = (x + 3)^2 = x^2 + 6x + 9 \Rightarrow 5x = 10 \Rightarrow x = 2$$

$$1257. 5^2 + x^2 = (10 - x)^2 = 100 - 20x + x^2 \Rightarrow 20x = 75 \Rightarrow x = \frac{15}{4} = 3.75 \text{ cm}$$

$$1258. a) 14^2 + (35 - x)^2 = (35 + x)^2 \Rightarrow 196 - 70x = 70x \Rightarrow x = \frac{196}{140} = \frac{14 \cdot 14}{10 \cdot 14} = 1.4 \text{ cm}$$

Svar: Konens höjd är 33.6 cm

$$b) V_{kon} = \frac{bh}{3} = \frac{3.36}{3} \pi 1.4^2 \approx 7 \text{ liter}$$

1312. Fall 1: 15 är en katet och 4x är hypotenusa.

$$15^2 + 4x^2 = 16x^2 \Rightarrow 12x^2 = 225 \Rightarrow x = \sqrt{\frac{225}{12}} \approx 4.3 \text{ cm}$$

Svar: 2x= 8.7 cm

Fall 2: 4x är en katet och 15 är hypotenusa.

$$15^2 = 4x^2 + 16x^2 \Rightarrow 20x^2 = 225 \Rightarrow x = \sqrt{\frac{225}{20}} \approx 3.4 \text{ cm}$$

Svar: 2x= 6.7 cm

$$1313. a) \frac{x}{3} = \frac{27}{x} \Rightarrow x^2 = 81 \Rightarrow x = \pm 9$$

$$b) 3x = (\sqrt{x} + 3)(\sqrt{x} - 3) + 10 = x - 9 + 10 = x + 1 \Rightarrow 2x = 1 \Rightarrow x = 0.5$$

$$1314. \frac{3x-2}{x-1} = \frac{x-1}{x} \Rightarrow (3x-2)x = (x-1)^2 \Rightarrow$$

$$3x^2 - 2x = x^2 - 2x + 1 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm \frac{1}{\sqrt{2}} = \pm \sqrt{0.5}$$

$$1315. (x + a)^2 = (2a - 3x)^2 \Rightarrow \begin{cases} x_1 + a = 2a - 3x_1 \\ x_2 + a = -2a + 3x_2 \end{cases} \Rightarrow \begin{cases} 4x_1 = a \\ 2x_2 = 3a \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = \frac{a}{4} = 0.25a \\ x_2 = \frac{3a}{2} = 1.5a \end{cases}$$

$$1316. (6 - a)^2 = 16 \Rightarrow 6 - a = \pm 4 \Rightarrow a = 6 \mp 4 \Rightarrow \begin{cases} a_1 = 2 \\ a_2 = 10 \end{cases} \Rightarrow$$

$$\begin{cases} (x-2)^2 = 16 \\ (x-10)^2 = 16 \end{cases} \Rightarrow \begin{cases} x_1 = 6, x_2 = -2 \\ x_3 = 14, x_4 = 6 \end{cases}$$

1322.

$$(x+6)^2 + x^2 = 20^2 \Rightarrow x^2 + 12x + 36 + x^2 = 400 \Rightarrow 2x^2 + 12x - 364 = 0 \Rightarrow$$

$$x^2 + 6x - 182 = 0 \Rightarrow x = -3 \pm \sqrt{9 + 182} \approx -3 \pm 13.8 = 10.8 \text{ cm}$$

$$A_{\Delta} = \frac{10.8(10.8 + 6)}{2} \approx 91 \text{ cm}^2$$

$$1323. y^4 - 10y^2 + 9 = 0 \Rightarrow y^2 = 5 \pm \sqrt{25 - 9} = 5 \pm 4 = \begin{cases} y_1^2 = 9 \\ y_2^2 = 1 \end{cases} \Rightarrow \begin{cases} y_1 = \pm 3 \\ y_2 = \pm 1 \end{cases}$$

$$1338. (x+1)(x-3) = 0 \Rightarrow x^2 - 3x + x - 3 = 0 \Rightarrow x^2 - 2x - 3 = 0$$

$$1339. 1.2 = 0.0001x^2 - 0.016x + 1.34 \Rightarrow 0.0001x^2 - 0.016x + 0.14 = 0 \Rightarrow$$

$$x^2 - 160x + 1400 = 0 \Rightarrow x = 80 \pm \sqrt{80^2 - 1400} \approx 150 \text{ km/h}$$

$$1340. \text{ a) } x^4 - 13x^2 + 36 = 0 \Rightarrow x^2 = 6.5 \pm \sqrt{6.5^2 - 36} = 6.5 \pm 2.5 = \begin{Bmatrix} 9 \\ 4 \end{Bmatrix} \Rightarrow$$

$$x^2 = \begin{Bmatrix} 9 \\ 4 \end{Bmatrix} \Rightarrow x = \begin{Bmatrix} \pm 3 \\ \pm 2 \end{Bmatrix}$$

$$\text{ b) } x - 5\sqrt{x} + 4 = 0 \Rightarrow \sqrt{x} = 2.5 \pm \sqrt{2.5^2 - 4} = \begin{Bmatrix} 4 \\ 1 \end{Bmatrix} \Rightarrow x = \begin{Bmatrix} 16 \\ 1 \end{Bmatrix}$$

$$1341. \text{ a) } ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \Rightarrow p = \frac{b}{a} \text{ och } q = \frac{c}{a}$$

$$\text{ b) } x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}} = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ VSB}$$

$$1352. \text{ a) } a = 5 \cdot 6, b = -11 \Rightarrow x^2 - 11x + 30 = 0$$

$$\text{ b) } \begin{cases} r_1 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q} \\ r_2 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q} \end{cases} \Rightarrow \begin{cases} r_1 + r_2 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q} - \frac{p}{2} - \sqrt{\frac{p^2}{4} - q} \\ r_1 r_2 = \left(-\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}\right) \left(-\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}\right) \end{cases}$$

$$\begin{cases} r_1 + r_2 = -p \\ r_1 r_2 = \frac{p^2}{4} - \left(\frac{p^2}{4} - q\right) = q \end{cases}$$

1362. a)

n	1	2	3	4	5	6	7	8	9	10	11	12
i^n	i	-1	$-i$	1	i	-1	$-i$	1	i	-1	$-i$	1

b) $i^n = 1$ när n är delbart med 4.

c) $i^{87} + i^{4001} = i^{84+3} + i^{4000+1} = i^3 + i^1 = -i + i = 0$

1363.

Lösning 1:

$$2z^2 + 2(4 - 2i)z - 16i = 0 \Rightarrow z^2 + (4 - 2i)z - 8i = 0$$

$$z^2 + 4z - 2iz - 8i = 0 \Rightarrow z(z + 4) - 2i(z + 4) = (z + 4)(z - 2i) = 0$$

Lösning 2: (med pq -formel, men mera tidskrävande)

$$z = -\frac{4 - 2i}{2} \pm \sqrt{\left(\frac{4 - 2i}{2}\right)^2 + 8i} = i - 2 \pm \sqrt{\frac{16 - 16i - 4}{4} + 8i} =$$

$$= i - 2 \pm \sqrt{3 - 4i + 8i} = i - 2 \pm \sqrt{3 + 4i} = i - 2 \pm (2 + i) = \begin{cases} 2i \\ -4 \end{cases}$$

$$1364. z_1 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}, z_2 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}, z_1 + z_2 = -p = 12,$$

$$z_1 = a + bi \text{ och } z_2 = a - bi \Rightarrow z_1 + z_2 = 2a = 12 \Rightarrow a = 6$$

Då rötterna är komplexa måste:

$$\frac{p^2}{4} - q < 0 \Rightarrow \frac{12^2}{4} - q < 0 \Rightarrow 36 - q < 0 \Rightarrow q > 36$$

$$1370. a) \sqrt{3 - x} = -x - 9 \Rightarrow 3 - x = x^2 + 18x + 81$$

$$\Rightarrow x^2 + 19x + 78 = 0 \Rightarrow x = -9.5 \pm \sqrt{9.5^2 - 78} = -9.5 \pm 3.5 = \begin{cases} x_1 = -6 \\ x_2 = -13 \end{cases}$$

$$\text{Test: } \begin{cases} \sqrt{3 - (-6)} = -(-6) - 9 \\ \sqrt{3 - (-13)} = -(-13) - 9 \end{cases} \Rightarrow \begin{cases} \sqrt{9} = -3 \text{ NEJ, falsk rot!} \\ \sqrt{16} = 4 \text{ Stämmer!} \end{cases}$$

$$b) \sqrt{x^2 + \sqrt{20x + 70}} = x + 2 \Rightarrow x^2 + \sqrt{20x + 70} = x^2 + 4x + 4 \Rightarrow$$

$$\Rightarrow \sqrt{20x + 70} = 4x + 4 \Rightarrow 20x + 70 = 16x^2 + 32x + 16 \Rightarrow$$

$$\Rightarrow 16x^2 + 12x - 54 = 0 \Rightarrow x^2 + \frac{3}{4}x - \frac{27}{8} = 0 \Rightarrow x = -\frac{3}{8} \pm \sqrt{\left(\frac{3}{8}\right)^2 + \frac{27}{8}}$$

$$\Rightarrow x = -\frac{3}{8} \pm \sqrt{\frac{225}{64}} = \frac{-3 \pm 15}{8} = \begin{cases} x_1 = \frac{3}{2} \text{ Riktig rot!} \\ x_2 = -\frac{9}{4} \text{ falsk rot.} \end{cases}$$

1371. A. $\sqrt{2x+5} = x+1 \Rightarrow 2x+5 = x^2+2x+1 \Rightarrow x = \pm 2$ men -2 är en falsk rot.

B. $\sqrt{2x+5} = -(x+1) \Rightarrow$ Bara $x = -2$ är lösning.

C. $2x+5 = (x+1)^2 \Rightarrow x = \pm 2$ båda är riktiga rötter.

1381.

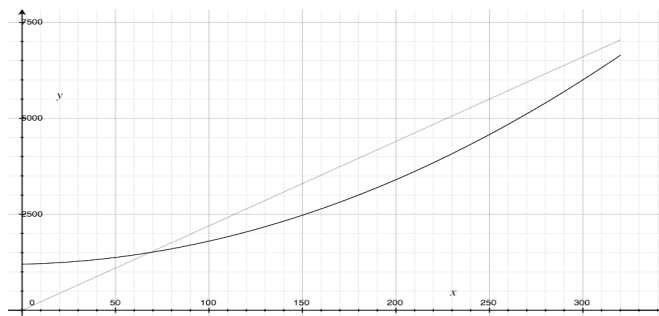
a) $6000 = 1200 + x + 0.05x^2 \Rightarrow x^2 + 20x - 96000 = 0 \Rightarrow$

$$x = -10 \pm \sqrt{100 + 96000} = 300 \text{ st (den negativa roten stryks)}$$

b) $22x = 1200 + x + 0.05x^2 \Rightarrow x^2 - 420x + 24000 = 0$

x

$$= 210 \pm \sqrt{210^2 - 24000} = \begin{cases} 68 \text{ flaskor} \\ 352 \end{cases} \text{ (utanför definitionsmängden)}$$



1382.

$$\begin{cases} a+b=51 \\ ab=144 \end{cases} \Rightarrow a + \frac{144}{a} = 51 \Rightarrow a^2 - 51a + 144 = 0 \Rightarrow a = 25.5 \pm \sqrt{25.5^2 - 144}$$

$$a = 25.5 \pm 22.5 = \begin{cases} 48 \\ 3 \end{cases} \Rightarrow \begin{cases} a_1 = 48 \\ b_1 = 3 \end{cases} \text{ eller } \begin{cases} a_2 = 3 \\ b_2 = 48 \end{cases}$$

1383. $h = r + 16, 105 = 2\pi r^2 + 2\pi r(r + 16) = 2\pi r^2 + 2\pi r^2 + 2\pi r16 \Rightarrow$

$$4\pi r^2 + 2\pi r16 - 105 = 0 \Rightarrow r^2 + \frac{2\pi 16}{4\pi} r - \frac{105}{4\pi} = 0 \Rightarrow r^2 + 8r - \frac{105}{4\pi} = 0 \Rightarrow$$

$$r = -4 \pm \sqrt{16 + \frac{105}{4\pi}} = 9.4 \text{ mm}$$

1384. Den lilla delen, $1-x$, förhåller sig till den stora som den stora till det hela.

$$\frac{1-x}{x} = \frac{x}{1} \Rightarrow 1-x = x^2 \Rightarrow x^2 + x - 1 = 0 \Rightarrow x = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 1}$$

$$\Rightarrow x = -\frac{1}{2} \pm \sqrt{\frac{1+4}{4}} = \{\text{ignorera den negativa lösningen}\} = \frac{\sqrt{5}-1}{2}$$

$$1412. a) x^3 - 81x = x(x^2 - 81) = x(x+9)(x-9)$$

$$\begin{aligned} b) 4(x+2)(x+7) - 8(x+2)(2x+3) &= 4(x+2)((x+7) - 2(2x+3)) = \\ &= 4(x+2)(x+7-4x-6) = 4(x+2)(1-3x) \end{aligned}$$

$$c) (x+3)^2 - 9y^2 = (x+3-3y)(x+3+3y)$$

$$d) b^{3n} - b^n = b^n(b^{2n} - 1) = b^n(b^n + 1)(b^n - 1)$$

$$1424. x^2 + a = 0 \Rightarrow (x+7)(x-7) = x^2 - 49 \Rightarrow a = -49$$

$$\begin{aligned} 1425. x^{80} - 1 &= (x^{40} + 1)(x^{40} - 1) = (x^{40} + 1)(x^{20} + 1)(x^{20} - 1) = \\ &= (x^{40} + 1)(x^{20} + 1)(x^{20} - 1) = (x^{40} + 1)(x^{20} + 1)(x^{10} + 1)(x^{10} - 1) = \\ &= (x^{40} + 1)(x^{20} + 1)(x^{10} + 1)(x^5 + 1)(x^5 - 1) \end{aligned}$$

$$\begin{aligned} 1439. z^5 - za^4 &= z(z^4 - a^4) = z(z^2 + a^2)(z^2 - a^2) = z(z^2 + a^2)(z+a)(z-a) \\ &= z(z^2 + 49)(z+7)(z-7) \Rightarrow z = 0, z = \pm 7 \text{ och } z = \pm 7i \end{aligned}$$

$$1440. f(3) = 3a(3+a) = 12 \Rightarrow a^2 + 3a - 4 = 0 \Rightarrow a = -1.5 \pm \sqrt{2.25 + 4} = \begin{Bmatrix} -4 \\ 1 \end{Bmatrix}$$

$$f(x) = \begin{cases} -4x(x-4) = 0 \Rightarrow x_1 = 0, x_2 = 4 \\ x(x+1) = 0 \Rightarrow x_3 = 0, x_4 = -1 \end{cases}$$

Test 1

$$1. a) (5-y)^2 = 25 - 10y + y^2$$

$$b) (1-y)^2 = 1 - 2y + y^2$$

$$c) (3y-2)^2 = 9y^2 - 12y + 4$$

$$2. (4x-3)(4x+3) + (x-9)^2 - 9(8-2x) =$$

$$= 16x^2 - 9 + x^2 - 18x + 81 - 72 + 18x = 17x^2$$

3.

$$(x-4)^2 + (x+1)(x+10) - 2x(1+x) = x^2 - 8x + 16 + x^2 + 11x + 10 - 2x - 2x^2 =$$

$$= x + 26$$

$$4. (x + 6)^2 + (x + 6)(x - 6) = x^2 + 12x + 36 + x^2 - 36 = 2x^2 + 12x$$

Simon har glömt den dubbla produkten $2 \cdot 6x$ i kvadreringen.

$$5. a) 2x(x - 2) = (x + 4)(2x - 6) \Leftrightarrow 2x^2 - 4x = 2x^2 + 2x - 24 \Rightarrow 24 = 6x \Rightarrow x = 4$$

$$b) (3t + 1)(t - 3) - 3(t + 2)(t - 2) = 0 \Leftrightarrow 3t^2 - 8t - 3 - 3t^2 + 12 = 0 \Rightarrow t = 9/8$$

$$6. a) 49x^2 = 36 \Leftrightarrow x^2 = \frac{36}{49} \Rightarrow x = \pm \frac{6}{7}$$

$$b) 81x^2 = 4 \Leftrightarrow x^2 = \frac{4}{81} \Rightarrow x = \pm \frac{2}{9}$$

$$7. a) 9x^2 - 64 = 0 \Leftrightarrow 9x^2 = 64 \Leftrightarrow x^2 = \frac{64}{9} \Rightarrow x = \pm \frac{8}{3}$$

$$b) 7 - 80x^2 = x^2 + 6 \Leftrightarrow 81x^2 = 1 \Leftrightarrow x^2 = \frac{1}{81} \Rightarrow x = \pm \frac{1}{9}$$

$$8. a) x^2 + x - 12 = 0 \Rightarrow x = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 12} = -\frac{1}{2} \pm \frac{7}{2} = \begin{cases} x_1 = 3 \\ x_2 = -4 \end{cases}$$

$$b) x^2 - 11x + 10 = 0 \Rightarrow x = \frac{11}{2} \pm \sqrt{\left(\frac{11}{2}\right)^2 - 10} = \frac{11}{2} \pm \frac{9}{2} = \begin{cases} x_1 = 10 \\ x_2 = 1 \end{cases}$$

$$9. a) x^2 - 6x - 16 = 0 \Rightarrow x = 3 \pm \sqrt{(3)^2 + 16} = 3 \pm 5 = \begin{cases} x_1 = 8 \\ x_2 = -2 \end{cases}$$

$$b) x^2 + 3x + 2 = 0 \Rightarrow x = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 2} = -1.5 \pm 0.5 = \begin{cases} x_1 = -1 \\ x_2 = -2 \end{cases}$$

$$10. a) z^2 + 12z + 38 = 0 \Rightarrow x = -6 \pm \sqrt{6^2 - 38} = -6 \pm i\sqrt{2}$$

$$b) 3z^2 - 6z + 15 = 0 \Rightarrow z^2 - 2z + 5 = 0 \Rightarrow x = 1 \pm \sqrt{1^2 - 5} = 1 \pm 2i$$

$$11. a) 9x^2 - 1 = \{\text{konjugatregeln}\} = (3x + 1)(3x - 1)$$

$$b) x^2 + 9 = \{\text{konjugatregeln}\} = (x + 3i)(x - 3i), \text{ går visst!}$$

$$c) x^2 - 16 = \{\text{konjugatregeln}\} = (x + 4)(x - 4)$$

$$12. a) x^3 - 2x^2 = x^2(x - 2)$$

$$b) 3xy + 5y^2 = y(3x + 5y)$$

$$c) xy^2 - xy = xy(y - 1)$$

$$13. a) 5x - 16x^2 + x^3 = x(x^2 - 16x + 5) = x(x - 8 + \sqrt{59})(x - 8 - \sqrt{59})$$

$$\begin{aligned} \text{b) } 4x^3 + 2x^2 - 4x^4 &= 2x^2(2x + 1 - 2x^2) = -4x^2 \left(x^2 - x - \frac{1}{2} \right) = \\ &= -4x^2 \left(x - \frac{1 - \sqrt{3}}{2} \right) \left(x - \frac{1 + \sqrt{3}}{2} \right) \end{aligned}$$

$$\text{c) } x^3 - x^4 = x^3(1 - x)$$

$$14. \text{ a) } x^3 - 4x^2 = x^2(x - 4) = 0 \Rightarrow x_1 = x_2 = 0 \text{ och } x_3 = 4$$

$$\text{b) } x^3 - 9x = x(x^2 - 9) = x(x + 3)(x - 3) = 0 \Rightarrow x_1 = 0, x_2 = 3 \text{ och } x_3 = -3$$

$$15. \text{ a) } (x - 5) \cdot 3x = 0 \Rightarrow x_1 = 5 \text{ och } x_2 = 0$$

$$\text{b) } (x - 2)(x + 7) = 0 \Rightarrow x_1 = 2 \text{ och } x_2 = -7$$

$$16. \text{ a) } 4(8 - 4x)(2x + 9)(3x - 1) = 0 \text{ nollställena hittas ett i varje faktor dvs}$$

$$x_1 = 2, x_2 = -4.5 \text{ och } x_3 = 1/3$$

$$\text{b) } x(2x + 7)(5x - 2) = 0, \text{ samma metod som i a) dvs}$$

$$x_1 = 0, x_2 = -3.5 \text{ och } x_3 = 0.4$$

$$17. \text{ a) } x^4 - 16x^2 = 0 \Leftrightarrow x^2(x^2 - 16) = x^2(x + 4)(x - 4) = 0$$

$$\begin{cases} x_1 = x_2 = 0 \\ x_3 = 4 \\ x_4 = -4 \end{cases}$$

$$\text{b) } x^4 = x^2 \Leftrightarrow x^2(x^2 - 1) = 0 \Rightarrow \begin{cases} x_1 = x_2 = 0 \\ x_3 = 1 \\ x_4 = -1 \end{cases}$$

$$18. x(x - a) = 3a \Leftrightarrow x^2 - ax - 3a = 0 \Rightarrow x = \frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 + 3a}$$

$$\frac{a^2}{4} + 3a < 0 \Leftrightarrow a^2 + 12a = a(a + 12) < 0 \Rightarrow -12 < a < 0$$

$$19. x^3 + x + (2x + 1)(x^2 + 1) = 0 \Leftrightarrow x(x^2 + 1) + (2x + 1)(x^2 + 1) = 0 \Leftrightarrow$$

$$(x^2 + 1)(3x + 1) = 0 \Rightarrow \begin{cases} x_1 = i \\ x_2 = -i \\ x_3 = -1/3 \end{cases}$$

$$20. z^2 + 2iz + p = 0 \Rightarrow z = -i \pm \sqrt{i^2 - p} = -i \pm \sqrt{-1 - p} \Rightarrow p = -5$$

$$z_2 = -i - 2$$

21. a)

$$\sqrt{x+6} = x \Leftrightarrow x+6 = x^2 \Leftrightarrow x^2 - x - 6 = 0 \Leftrightarrow x = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 6} = \frac{1 \pm 5}{2}$$

$$\begin{cases} x_1 = 3 \\ x_2 = -2, \text{ falsk rot} \end{cases}$$

b) $\sqrt{x+6} = 10x \Leftrightarrow x+6 = 100x^2 \Leftrightarrow x^2 - 0.01x - 0.06 = 0$

$$x = \frac{1}{200} \pm \sqrt{\left(\frac{1}{200}\right)^2 + \frac{6}{100}} = \frac{1}{200} \pm \sqrt{\frac{1}{400} + \frac{6 \cdot 4 \cdot 100}{4 \cdot 100 \cdot 100}} = \frac{1 \pm 49}{200}$$

$$\begin{cases} x_1 = \frac{1}{4} \\ x_2 = -\frac{6}{25}, \text{ falsk rot} \end{cases}$$

22. $x(x+12) = 189 \Leftrightarrow x^2 + 12x - 189 = 0 \Rightarrow x = -6 \pm \sqrt{36 - 189} = -6 \pm 15$

$$x = 9 \Rightarrow O = 2 \cdot (9 + 21) = 60 \text{ cm}$$

23. a) $200 - 7000h^2 = 100h \Leftrightarrow h^2 + \frac{1}{70}h - \frac{2}{70} = 0 \Rightarrow h = -\frac{1}{140} \pm \sqrt{\left(\frac{1}{140}\right)^2 + \frac{2}{70}} =$

$$= -\frac{1}{140} \pm \sqrt{\left(\frac{1}{140}\right)^2 + \frac{2 \cdot 2 \cdot 140}{70 \cdot 2 \cdot 140}} = \frac{-1 \pm 23.7}{140} = \begin{cases} h_1 = -0.176 \\ h_2 = 0.162 \end{cases}$$

b) $0.049h^2 + 6.5h + 100 = 0 \Rightarrow h^2 + \frac{6.5}{0.049}h + \frac{100}{0.049} = 0 \Rightarrow$

$$h = -\frac{6.5}{0.098} \pm \sqrt{\left(\frac{6.5}{0.098}\right)^2 - \frac{100}{0.049}} = \begin{cases} h_1 = -115 \\ h_2 = -17.8 \end{cases}$$

24. $\begin{cases} a \cdot b = 3847102 \\ a - b = 39 \end{cases} \Rightarrow a - \frac{3847102}{a} = 39 \Rightarrow a^2 - 39a - 3847102 = 0$

$$a = 19.5 \pm \sqrt{19.5^2 + 3847102} = \begin{cases} a_1 = 1981 \\ (a_2 = -1942) \end{cases}, b = 1942$$

Magnus är född 1942.

25. $x^2 + (x+10)^2 = 1.25^2(x+10)^2 \Rightarrow x^2 = (1.25^2 - 1)(x+10)^2 \Rightarrow$

$$x^2 = \left(\frac{25}{16} - 1\right)(x+10)^2 = \frac{9}{16}(x+10)^2 \Rightarrow x = \pm \frac{3}{4}(x+10) \Rightarrow$$

$$\Rightarrow \begin{cases} 4x = 3x + 30 \\ -4x = 3x + 30 \end{cases} \Rightarrow \begin{cases} x = 30 \\ -7x = 30 \text{ falsk rot} \end{cases} \quad O = 120 \text{ m}$$

Blandade uppgifter i kapitel 1

1. a) $(x + 6)(y + 2) - 2(x + 3y) = xy + 2x + 6y + 12 - 2x - 6y = xy + 12$

b) $(x + 3)^2 - 3(2x + 3) = x^2 + 6x + 9 - 6x - 9 = x^2$

2. a) $(x - 4)^2 + (4x + 1)^2 - 17 = x^2 - 8x + 16 + 16x^2 + 8x + 1 - 17 = 17x^2$

b) $4 - (3x - 2)^2 + 9x^2 = 4 - 9x^2 + 12x - 4 + 9x^2 = 12x$

3. a) $x^2 - 4 = (x + 2)(x - 2)$

b) $x^2 - 25 = (x + 5)(x - 5)$

c) $49 - x^2 = (7 + x)(7 - x)$

4. a) $x^4 - 9x^2 = x^2(x + 3)(x - 3)$

b) $y - y^3 = y(1 + y)(1 - y)$

c) $yx^2 - y = y(x + 1)(x - 1)$

5 a) $x^2 + 24^2 = x^2 + 32x + 16^2 \Leftrightarrow 32x = 24^2 - 16^2 \Rightarrow x = 10 \text{ m}$

b) $85^2 = (x - 8)^2 + (x + 9)^2 \Leftrightarrow 85^2 = x^2 - 16x + 64 + x^2 + 18x + 81$

$$2x^2 + 2x + 145 - 85^2 = 0 \Leftrightarrow x^2 + x - 3540 \Rightarrow x = -0.5 \pm \sqrt{0.5^2 + 3540} = 59 \text{ m}$$

Svar: Kortast sidan är 51 m

6. $6.5^2 = x^2 + (x - 3.5)^2 \Leftrightarrow 6.5^2 = 2x^2 - 7x + 3.5^2 \Leftrightarrow 2x^2 - 7x + 3.5^2 - 6.5^2 = 0 \Leftrightarrow$

$$x^2 - 3.5x - 15 = 0 \Rightarrow x = 1.75 \pm \sqrt{1.75^2 + 15} = 6, \text{ Svar: } 2.5 \text{ cm}$$

7. $x(x - 4) + 8 = x^2 - 6(x - 1) \Leftrightarrow x^2 - 4x + 8 = x^2 - 6x + 6 \Rightarrow 2x = -2 \Rightarrow x = -1$

8. $(x - 4)(3x - 2) - (3x + 1)(x - 5) = x \Leftrightarrow$

$$3x^2 - 14x + 8 - (3x^2 - 14x - 5) = x \Rightarrow 13 = x$$

9. a) $(7 - x)^2 + (14 - x)(7 - 2x) = (21 - x)(7 - 3x) \Leftrightarrow$

$$49 - 14x + x^2 + 98 - 35x + 2x^2 = 147 - 70x + 3x^2 \Leftrightarrow$$

$$-49x = -70x \Rightarrow x = 0$$

$$10. a) 3t^2 + 5 = t^2 + 9.5 \Leftrightarrow t^2 = 2.25 = t = \pm 1.5$$

$$b) (x - 20)^2 = 300 \Rightarrow x - 20 = \pm \sqrt{300} \Rightarrow x = \begin{cases} x_1 = 20 + \sqrt{300} \approx 37.3 \\ x_2 = 20 - \sqrt{300} \approx 2.68 \end{cases}$$

$$11. a) y^2 + 2y = 3 \Leftrightarrow y^2 + 2y - 3 = 0 \Rightarrow y = -1 \pm \sqrt{1+3} = \begin{cases} y_1 = 1 \\ y_2 = -3 \end{cases}$$

$$b) y^2 - 6y - 16 = 0 \Rightarrow y = 3 \pm \sqrt{3^2 + 16} = 3 \pm 5 = \begin{cases} 8 \\ -2 \end{cases}$$

$$12. a) 3z^2 - 6z + 6 = 0 \Leftrightarrow z^2 - 2z + 2 = 0 \Rightarrow z = 1 \pm \sqrt{1-2} = 1 \pm i$$

$$b) 1 - x = x^2 \Leftrightarrow x^2 + x - 1 = 0 \Rightarrow z = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 1} = \frac{-1 \pm \sqrt{5}}{2}$$

$$13. a) 5z^2 + 4z + 4 = 0 \Leftrightarrow z^2 + 0.8z + 0.8 = 0 \Rightarrow z = -0.4 \pm \sqrt{0.16 - 0.8} = -0.4 \pm 0.8i$$

$$b) (x + 4)^2 + (x - 6)^2 = 68 \Leftrightarrow x^2 + 8x + 16 + x^2 - 12x + 36 = 68$$

$$2x^2 - 4x - 16 = 0 \Leftrightarrow x^2 - 2x - 8 = 0 \Rightarrow x = 1 \pm \sqrt{1+8} = \begin{cases} x_1 = 4 \\ x_2 = -2 \end{cases}$$

14.

$$\begin{cases} S + J + C = 7225 \\ S = \frac{J}{3} \\ C = \frac{S}{4} \end{cases} \Rightarrow \frac{J}{3} + J + \frac{J}{12} = 7225 \Rightarrow \begin{cases} J = 5\,100 \text{ kg} \\ S = 1\,700 \text{ kg} \\ C = 425 \text{ kg} \end{cases}$$

$$15. x + 1.35 \cdot x = 265\,000 \Leftrightarrow 2.35x = 265\,000 \Rightarrow x \approx 113\,00 \text{ kunder}$$

$$16. x^2 - 2ax - 15a^2 = 0 \Rightarrow x = a \pm \sqrt{a^2 + 15a^2} = a \pm 4a = \begin{cases} 5a \\ -3a \end{cases} \text{ dvs } \begin{cases} a_1 = 8 \\ a_2 = -40/3 \end{cases}$$

$$17. a) \pi r^2 + 48\pi r - 2280 = 0 \Leftrightarrow r^2 + 2r - \frac{2280}{\pi} = 0 \Rightarrow r = -24 \pm \sqrt{24^2 + \frac{2280}{\pi}}$$

$$r \approx 12.1$$

$$b) 0.8y - \frac{1}{y-0.07} = 1 \Leftrightarrow 0.8y(y - 0.07) - 1 = y - 0.07 \Leftrightarrow 0.8y^2 - 0.056 - 1 = y - 0.07$$

$$0.8y^2 - y - 0.986 = 0 \Leftrightarrow y^2 - 1.25y - 1.2325 \Rightarrow$$

$$y = \frac{1.25}{2} \pm \sqrt{\left(\frac{1.25}{2}\right)^2 + 1.2325} \approx 1.90$$

$$18. (x - 3 + 5i)(x - 3 - 5i) = x^2 - 3x - 5ix - 3x + 9 + 15i + 5ix - 15i + 25 =$$

$$= x^2 - 6x + 34 = 0$$

$$19. (x+10)^2 = (x+3)^2 + (x-4)^2 \Leftrightarrow x^2 + 20x + 100 = x^2 + 6x + 9 + x^2 - 8x + 16$$

$$x^2 - 22x - 75 = 0 \Rightarrow x = 11 \pm \sqrt{11^2 + 75} = 25$$

$$\frac{(x+3)(x-4)}{2} = \frac{28 \cdot 21}{2} = 294 \text{ m}^2$$

$$\begin{aligned} 20. 10(x-0.2)^2 - 100(0.3x+5)(0.3x-5) - (x-50)^2 &= \\ &= 10x^2 - 4x + 0.4 - 100(0.09x^2 - 25) - x^2 + 100x - 2500 = \\ &= 0.4 + 96x \end{aligned}$$

$$21. (\sqrt{3} - \sqrt{12})^2 = 3 - 2\sqrt{3}\sqrt{12} + 12 = 15 - 2\sqrt{3 \cdot 12} = 3$$

$$22. 2z^3 + 20z^2 + 82z = 2z(z^2 + 10z + 41) = 0 \Rightarrow z = -5 \pm \sqrt{25 - 41} = \begin{cases} z_1 = 0 \\ z_2 = -5 + 4i \\ z_3 = -5 - 4i \end{cases}$$

$$23. 24^2 + 6370^2 = (6370 + h)^2 \Rightarrow h = \sqrt{24^2 + 6370^2} - 6370 \approx 45 \text{ m}$$

$$24. x(2x - a) = 0 \Rightarrow a = 10$$

$$25. ((x+3)^2 - (x-3)^2)^2 = (x^2 + 6x + 9 - x^2 + 6x - 9)^2 = (12x)^2 = 144x^2$$

$$26. (7+2i)(7-2i) = 49 - 14i + 14i - (2i)^2 = 49 - (-4) = 53$$

$$\begin{aligned} 27. 4^x + 1 + (2^x + 2^{2x})^2 - (4^x + 1)^2 &= \\ &= 4^x + 1 + 2^{2x} + 2 \cdot 2^x \cdot 2^{2x} + 2^{4x} - 4^{2x} - 2 \cdot 4^x - 1 = \\ &= 2 \cdot 2^x \cdot 2^{2x} = 2^{3x+1} \end{aligned}$$

$$28. 13^2 = x^2 + (x+7.2)^2 = x^2 + x^2 + 14.4x + 7.2^2 \Rightarrow x^2 + 7.2x + \frac{7.2^2 - 13^2}{2} = 0 \Rightarrow$$

$$x = -\frac{7.2}{2} \pm \sqrt{\left(\frac{7.2}{2}\right)^2 - \frac{7.2^2 - 13^2}{2}} \approx 4.9 \text{ cm}$$

$$29. \text{ a) } 2x^3 - 5x^2 = x^2(2x - 5) = 0 \Rightarrow x_1 = x_2 = 0 \text{ och } x_3 = 2.5$$

$$\text{ b) } x(x^2 - 4) + 5(x^2 - 4) = (x^2 - 4)(x + 5) = 0 \Rightarrow x_1 = 2, x_2 = -2 \text{ och } x_3 = -5$$

$$30. 8^2 + x^2 = (40 - 8 - x)^2 \Rightarrow 64 + x^2 = 32^2 - 64x + x^2 \Rightarrow 64x = 960 \Rightarrow x = 15$$

Hypotenusan är: 17 cm

31.

$$\frac{x^2}{0.0050 - x} = 1.5 \cdot 10^{-4} \Rightarrow x^2 + 1.5 \cdot 10^{-4}x - 7.5 \cdot 10^{-7} \Rightarrow$$

$$\Rightarrow x = -\frac{1.5 \cdot 10^{-4}}{2} + \sqrt{\left(\frac{1.5 \cdot 10^{-4}}{2}\right)^2 + 7.5 \cdot 10^{-7}} \approx 7.94 \cdot 10^{-4}$$

$$32. (x-2)(x^8-81) = (x-2)(x^4+9)(x^4-9) = (x-2)(x^4+9)(x^2+3)(x^2-3) \Rightarrow$$

$$x_1 x_2 x_3 = 2(-\sqrt{3})\sqrt{3} = -6$$

$$33. \text{Standard: } x^2 + \left(\frac{4}{3}\right)^2 x^2 = 28^2 \Rightarrow \frac{25}{9}x^2 = 28^2 \Rightarrow x = \sqrt{\frac{9 \cdot 28^2}{25}} \Rightarrow A = \frac{4}{3} \frac{9 \cdot 28^2}{25} \approx 376 \text{ tum}^2$$

$$\text{Bredbild: } y^2 + \left(\frac{16}{9}\right)^2 y^2 = 28^2 \Rightarrow \frac{337}{81}y^2 = 28^2 \Rightarrow y = \sqrt{\frac{81 \cdot 28^2}{337}} \Rightarrow A = \frac{16}{9} \frac{81 \cdot 28^2}{337} \approx 335 \text{ tum}^2$$

$$34. \text{a) } x \cdot 1.25x = x + 1.25x \Rightarrow x \cdot 1.25 = 2.25 \Rightarrow x = 1.8 \text{ och } 2.25$$

$$\text{b) } 4x \cdot 5x = 2x \cdot 3x + 4046 \Rightarrow 14x^2 = 4046 \Rightarrow x = 17$$

Talen är: 34, 51, 68, 85 dvs summan är 238.

$$35. x(32-x) + (x+2)(30-x) = 510$$

$$32x - x^2 - x^2 + 28x + 60 = 510 \Rightarrow x^2 - 30x + 225 = 0$$

$$x = 15 \pm \sqrt{15^2 - 225} = 15 \text{ cm}$$

36.

$$(x+1)^3 = x^3 + 1 \Rightarrow x^3 + 3x^2 + 3x + 1 = x^3 + 1 \Rightarrow 3x(x+1) = 0 \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -1 \end{cases}$$

$$37. (x+a)(x-4) = a(x-2) \Rightarrow x^2 - 4x + ax - 4a = ax - 2a \Rightarrow$$

$$x^2 - 4x - 2a = 0 \Rightarrow x = 2 \pm \sqrt{4 + 2a} \Rightarrow a = -2 \text{ ger dubbelrot}$$

$$38. (x+2+3y)^2 - (x+3y)^2 = (x+2+3y+x+3y)(x+2+3y-(x+3y)) =$$

$$= (2x+2+6y)(x+2+3y-x-3y) = 2(x+1+3y)2$$

Är jämnt delbart med 4 om x och y är hela tal.

$$40. ax^2 + 25a = 3x \Rightarrow x^2 - \frac{3}{a}x + 25 = 0 \Rightarrow x = \frac{3}{2a} \pm \sqrt{\frac{9}{4a^2} - 25} \Rightarrow \frac{9}{4a^2} - 25 < 0$$

$$\Rightarrow \frac{9}{4a^2} < 25 \Rightarrow \frac{9}{100} < a^2 \Rightarrow \frac{3}{10} < |a|$$

41. Kalla vattnets djup för x . Då gäller $(x + 1.5)^2 = x^2 + (2\sqrt{2})^2$

$$x^2 + 3x + 1.5^2 = x^2 + 8 \Rightarrow x = \frac{8 - 2.25}{3} \approx 1.9 \text{ m}$$

$$42. x^2 - 15ax + 9 = 0 \Rightarrow x = 7.5a \pm \sqrt{(7.5a)^2 - 9} \Rightarrow (7.5a)^2 - 9 > 0 \Rightarrow$$

$$(7.5a)^2 > 9 \Rightarrow a^2 > 0.16 \Rightarrow |a| > 0.4$$

$$43. (x^2 - 2x)^{1/2} = \sqrt{x^2 - 2x} = \sqrt{-5} \Rightarrow x^2 - 2x + 5 = 0 \Rightarrow x = 1 \pm \sqrt{1 - 5} \Rightarrow$$

$$x = 1 \pm \sqrt{-4} = 1 \pm 2i \Rightarrow (1 + 2i)^2 + (1 - 2i)^2 = 1 + 4i - 4 + 1 - 4i - 4 = \\ = 1 - 4 + 1 - 4 = -6$$

$$44. x_1^2 + x_2^2 = 45 \text{ och } \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 x_2} = \frac{1}{2} \Rightarrow 4(x_1 + x_2) = 2x_1 x_2 \Rightarrow$$

$$x_1^2 + x_2^2 + 2x_1 x_2 = 45 + 4(x_1 + x_2) \Rightarrow (x_1 + x_2)^2 = 45 + 4(x_1 + x_2)$$

$$\Rightarrow (-p)^2 = 45 + 4(-p) \Rightarrow p^2 + 4p - 45 = 0 \Rightarrow p = -2 \pm \sqrt{4 + 45}$$

$$\Rightarrow p = -2 \pm 7 = \begin{Bmatrix} 5 \\ -9 \end{Bmatrix} \text{ och } -4p = \{\text{se uppgift 1352}\} = 2q \Rightarrow q = -2p = \begin{Bmatrix} -10 \\ 18 \end{Bmatrix}$$

$$x^2 + 5x - 10 = 0 \text{ eller } x^2 - 9x + 18 = 0$$

45. Kalle får 1 ± 3 med $q_{fel} \Rightarrow p_{rätt} = -2$

Pelle får -1 ∓ 2 med p_{fel} .

Använder vi Pelles svar för att hitta $q_{rätt}$ fås:

$$4 = \left(\frac{-2}{2}\right)^2 - q_{rätt} \Rightarrow q_{rätt} = -3 \text{ Svar: } x^2 - 2x - 3 = 0$$

$$\begin{aligned}
&(((m \cdot 100 + d) \cdot 2 + 5)10 + 23)5 + \text{ålder} = \\
&((m \cdot 100 + d) \cdot 2 + 5)50 + 115 + \text{ålder} = \\
&(m \cdot 100 + d) \cdot 100 + 250 + 115 + \text{ålder} = \\
&(m \cdot 100 + d) \cdot 100 + 365 + \text{ålder} = \\
&m \cdot 100 \cdot 100 + d \cdot 100 + 365 + \text{ålder} = \\
&m \cdot 10000 + d \cdot 100 + 365 + \text{ålder} =
\end{aligned}$$

Subtraheras 365 kommer åldern att ockupera de första två siffrorna till höger, dagen den tredje och eventuellt fjärde från höger, kvar den femte och eventuellt den sjätte till månaden.

VSV