Valda uppgifter i kursboken Matematik M3c av Sjunnesson med flera utgiven på Liber, (2012).

4114. $f'(x) = 12e^{2x} \Rightarrow f(x) = 6e^{2x} + C,$ men $f(0) = 30 \Rightarrow f(x) = 6e^{2x} + 24$

4115. $f(x) = e^x \Rightarrow f'(x) = e^x$, punkten på kurvan är (a, e^a) och lutningen e^a dvs tangenten beskrivs enlig enpunktsformeln av $y - e^a = e^a(x - a)$. y-axeln skärs när $y = 0$ dvs $-e^a = e^a(x - a)$ eller $x = a - 1$ VSV.

4116.

A: $f(2a) = e^{2a} = (e^a)^2 \neq ae^a$

B: $f(2a) = e^{2a} = (e^a)^2 = \left(f(a)\right)^2$

C: $f(2a) = e^{2a} = (e^a)^2 \neq e^{2+a} = e^2e^a = f(2)f(a)$

D: $f(2 + a) = e^{2+a} = e^2e^a = f(2)f(a)$

E: $f(-a) = e^{-a} = \frac{1}{f(a)} \neq -f(a)$

F: $f(-a) = e^{-a} = \frac{1}{f(a)} = \left(f(a)\right)^{-1}$

4128. Det beror på om *k* är större än eller mindre än 1. Om $k > 1$ så är derivatan den röda. Annars tvärtom.

4129. $f(x) = x^2 + e^{2x} \Rightarrow f'(x) = 2x + 2e^{2x} = 0$ då $e^{2x} = -x \Rightarrow x \approx -0.426$

4130. Skillnaden är $d = e^x - 2x$ derivatan är $e^x - 2 = 0 \Rightarrow x = \ln 2 \Rightarrow d = 2 - 2\ln 2$

4131. a) $y(t) = 130 \cdot 1.2^x = 130 \cdot e^{xln1.2}$

b) $y(10) = 130 \cdot e^{10ln1.2} \approx 805°C$

c) $y'(t) = 130 \cdot \ln 1.2\, e^{xln1.2}$

d) $y'(10) = 130 \cdot \ln 1.2\, e^{10ln1.2} \approx 147$ °/min

4132. En inflexionspunkt är där andraderivatan byter tecken. y-axeln skärs då $x = 0$.

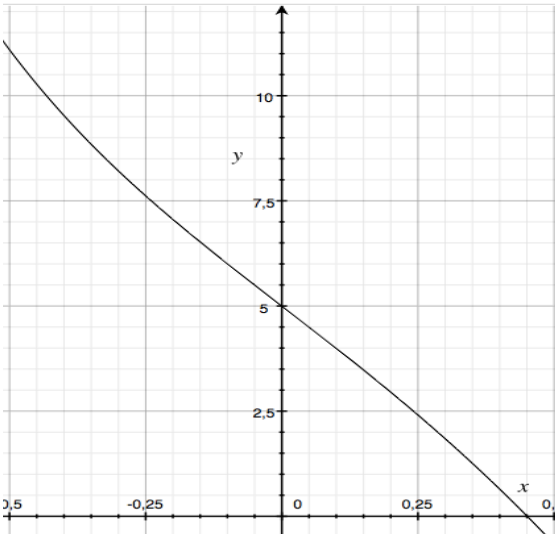
$$f(x) = 5e^{-2x} + a^2 - 10x^2 \Rightarrow f'(x) = -10e^{-2x} - 20x \Rightarrow f''(x) = 20e^{-2x} - 20 =$$

$$= 20(e^{-2x} - 1)$$

Ett teckenschema visar:

x	-0.1	0	0.1
$f''(x)$	$+$	0	$-$

Dvs en inflexionspunkt.



4140. $y(x) = 15\,000 \cdot 1.06^x = 15\,000 \cdot e^{x \ln 1.06} \Rightarrow y'(x) = 15\,000 \cdot \ln 1.06 \cdot e^{x \ln 1.06}$

$$15\,000 \cdot \ln 1.06 \cdot e^{x \ln 1.06} = 1200 \Rightarrow x \cdot \ln 1.06 = \ln \frac{1200}{15\,000 \cdot \ln 1.06} \Rightarrow x \approx 5.4$$

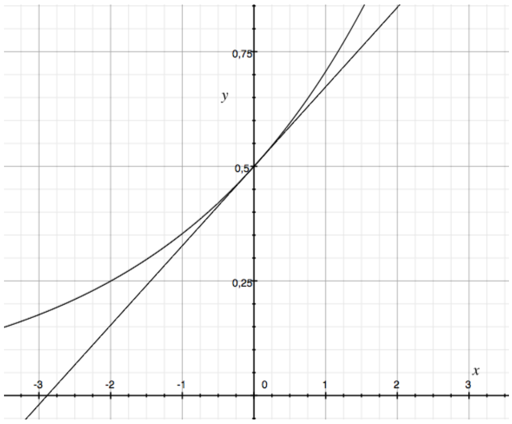
Vid tiden 5.4 år är tillväxten 1200 kr/år

4141. $g(x) = C \cdot 2^{0.5x}$ men $g(4) = C \cdot 2^{0.5 \cdot 4} = 2 \Rightarrow C = 0.5$ dvs $g(x) = \frac{1}{2} \cdot e^{x 0.5 \ln 2} \Rightarrow$

$$g'(x) = \frac{1}{2} \cdot 0.5 \ln 2\, e^{x 0.5 \ln 2} \Rightarrow g'(0) = \frac{1}{2} \cdot 0.5 \ln 2\, e^{0 \cdot 0.5 \ln 2} = \frac{1}{4} \ln 2$$

Tangenten får ekvationen $y - 0.5 = \frac{1}{4} \ln 2\, (x - 0)$ vilken skär *x*-axeln då $y = 0$ dvs då

$$x = -\frac{2}{\ln 2} \approx -2.9$$



4153. a)

$$T(t) = t_0 e^{-kt}, T'(t) = -kt_0 e^{-kt} \Rightarrow \begin{cases} 76 = t_0 e^{-k4} \\ -4.1 = -kt_0 e^{-k4} \end{cases} \Rightarrow 76k = 4.1 \Rightarrow$$

$$\Rightarrow \begin{cases} k = \dfrac{4.1}{76} \approx 0.054 \\ t_0 = 76 e^{4 \cdot \frac{4.1}{76}} \approx 94° \end{cases} \Rightarrow T(t) = 94e^{-0.054t}$$

b)

$$94e^{-0.054t} = 55 \Rightarrow t \approx 10 \text{ h}$$

4154.

$$h_1 = 13.4 \ln d - 21.4, h_2 = 13.4 \ln 2d - 21.4 \Rightarrow$$

$$h_2 - h_1 = 13.4 \ln 2d - 21.4 - (13.4 \ln d - 21.4) =$$

$$= 13.4(\ln d + \ln 2) - 13.4 \ln d = 13.4 \ln 2 \approx 9.3 \text{ m}$$

4155.

$$M(x) = M_0 \cdot e^{-\lambda x}, M(T) = \frac{M_0}{2} \text{ då } e^{-\lambda T} = \frac{1}{2} \Rightarrow \lambda T = \ln 2 \Rightarrow T = \frac{\ln 2}{\lambda}$$

4156.

$$y(x) = 200 - 180 \cdot e^{-kx}$$

a) $y(0) = 200 - 180 = 20°$

b)

$$\lim_{x\to\infty} y(x) = \lim_{x\to\infty} (200 - 180 \cdot e^{-kx}) = 200°$$

c) $y'(x) = -180(-k) \cdot e^{-kx} \Rightarrow y'(0) = 180k = 2.4 \Rightarrow k \approx 0.013 \Rightarrow$

$$y(18) = 200 - 180 \cdot e^{-0.013 \cdot 18} \approx 58°$$

d)

$$y(t) = 200 - 180 \cdot e^{-0.013 \cdot t} = 65 \Rightarrow t \approx 22 \text{ min}$$

4157.

$$y = Ce^{-2x} \Rightarrow y' = -2Ce^{-2x} \Rightarrow y'' = 4Ce^{-2x}$$

Sätt in dessa i uttrycket:

$$y'' + y' - 2y = 4Ce^{-2x} + (-2Ce^{-2x}) - 2(Ce^{-2x}) = 0 \text{ VSV}$$

Test 4

1. a)

$$f'(x) = 3e^x + 12e^{-3x}$$

b)

$$f(x) = 100 \cdot 1.03^x = 100 \cdot e^{x \ln 1.03} \Rightarrow f'(x) = 100 \cdot \ln 1.03 \cdot 1.03^x$$

2. a)

$$\ln x = 2 \Rightarrow x = e^2$$

b)

$$3e^x = 12 \Rightarrow e^x = 4 \Rightarrow x = \ln 4$$

3. a)

$$f(x) = 6x^2 - 4x^3 \Rightarrow F(x) = 2x^3 - x^4 + C$$

b)

$$f(x) = 2e^{x/3} \Rightarrow F(x) = 6e^{x/3} + C$$

4. a)

$$f(x) = \frac{3}{\sqrt{x}} \Rightarrow F(x) = 6\sqrt{x}$$

b)

$$g(x) = \frac{5}{x^2} \Rightarrow F(x) = -\frac{5}{x}$$

5. a)

$$f(x) = 3e^x - e^{-x/2} \Rightarrow F(x) = 3e^x + 2e^{-x/2} + C, F(0) = 3 + 2 + C = 1 \Rightarrow$$

$$F(x) = 3e^x + 2e^{-x/2} - 4$$

b)

$$f(x) = 3x^2 + 2x - 3 \Rightarrow F(x) = x^3 + x^2 - 3x + C, F(1) = 1 + 1 - 3 + C = 4 \Rightarrow$$

$$F(x) = x^3 + x^2 - 3x + 5$$

6.

$$v(t) = 5t \Rightarrow s(t) = 2.5t^2 + C, s(2) = 2.5 \cdot 2^2 + C = 6 \Rightarrow C = -4$$

$$s(t) = 2.5t^2 - 4 \text{ m}$$

7. a)

$$\int_0^3 4dx = 4[x]_0^3 = 4(3-0) = 12$$

b)

$$\int_2^6 (3t^2 - 2t + 1)dt = [t^3 - t^2 + t]_2^6 = 6^3 - 6^2 + 6 - (2^3 - 2^2 + 2) = 180$$

c)

$$\int_{-5}^0 2e^{-0.2x}dx = 10[e^{-0.2x}]_0^{-5} = 10(e-1)$$

10.

$$\int_1^3 (4x^7 - 2x^3 - 2x + 2)dx + \int_1^3 (-4x^7 + 2x^3 + 2x + 2)dx =$$

$$= \int_1^3 (4x^7 - 4x^7 - 2x^3 + 2x^3 - 2x + 2x + 2 + 2)dx =$$

$$= \int_1^3 4dx = 4[x]_1^3 = 4(3-1) = 8$$

11.

$$\int_0^{30} f(t)dt = 240 \text{ m}^3$$

Betyder att under 30 dagar förbrukades 240 m^3 vatten.

12.

$$g(t) = (e^t + 3)^2 = e^{2t} + 6e^t + 9 \Rightarrow G(t) = \frac{1}{2}e^{2t} + 6e^t + 9t + C \Rightarrow$$

$$G(0) = \frac{1}{2}e^0 + 6e^0 + 9 \cdot 0 + C = 3.5 \Rightarrow C = -3$$

$$G(t) = \frac{1}{2}e^{2t} + 6e^t + 9t - 3$$

13.

$$\int_1^a 2xdx = [x^2]_1^a = a^2 - 1 = 8 \Rightarrow a = 3$$

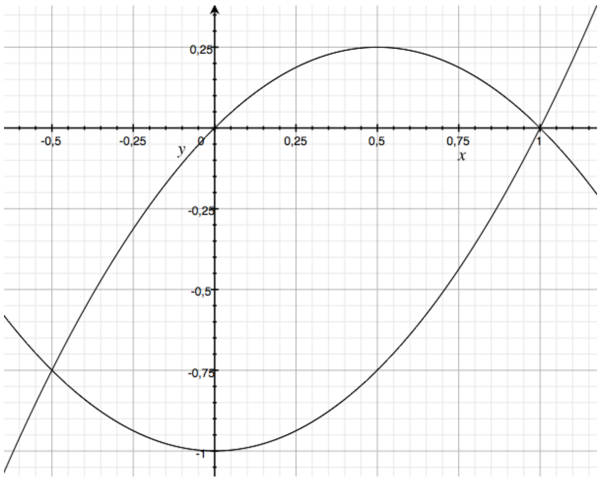
14.
a) ur grafen läses direkt att $F(2) = 5$

b) ur grafen fås direkt att $F(2) - F(0) = 5 - 1 = 4$

c)

$$\int_0^6 f(x)dx = [F(x)]_0^6 = F[6] - F[0] = 1 - 1 = 0$$

15.



$$\int_{-0.5}^{1} (x - x^2 - x^2 + 1)dx = \int_{-0.5}^{1} (x - 2x^2 + 1)dx = \left[\frac{1}{2}x^2 - \frac{2}{3}x^3 + x\right]_{-0.5}^{1} =$$

$$= \frac{1}{2}1^2 - \frac{2}{3}1^3 + 1 - \left(\frac{1}{2}(-0.5)^2 - \frac{2}{3}(-0.5)^3 - 0.5\right) =$$

$$= \frac{1}{2} - \frac{2}{3} + 1 - \left(\frac{1}{2}0.25 + \frac{2}{3}0.125 - 0.5\right) =$$

$$= \frac{12}{24} - \frac{16}{24} + \frac{24}{24} - \frac{3}{24} - \frac{2}{24} + \frac{12}{24} = \frac{27}{24} = \frac{9}{8} = 1.125$$

16. a)

$$\int e^x dx = e^x + C$$

$$\int_0^3 f(x)dx = e^3 - 1, \int_3^5 f(x)dx = e^5 - e^3 \text{ och } \int_0^5 f(x)dx = e^5 - 1$$

b)

$$\int_0^5 f(x)dx = \int_0^3 f(x)dx + \int_3^5 f(x)dx$$

17. a)

$$y = C \cdot a^x = 2000 \cdot 1.04^x$$

b)

$$y = C \cdot e^x = 2000 \cdot e^{x \ln 1.04}$$

c)

$$y'^{(5)} = 2000 \cdot \ln 1.04 \cdot e^{x \ln 1.04} \approx 95 \text{ kr/år}$$

Kapitaltillväxten efter 5 år är cirka 95 kr/år.

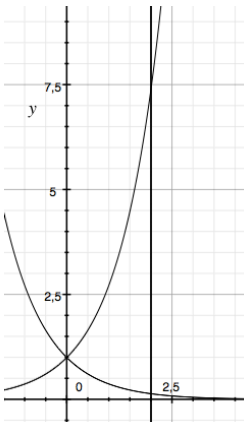
18. a)

$$\int_0^2 3x^2 dx = [x^3]_0^2 = 3^2 - 0^3 = 8 \text{ a. e.}$$

b)

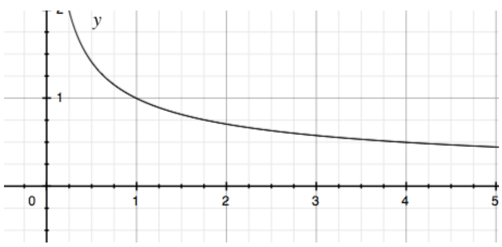
$$\int_0^1 0.5e^{2x} dx = [0.25e^{2x}]_0^1 = 0.25(e^2 - 1) \approx 1.6 \text{ a. e.}$$

19.



$$\int_0^2 (e^x - e^{-x})dx = [e^x + e^{-x}]_0^2 = e^2 + e^{-2} - 2 \approx 5.5 \text{ a. e.}$$

20.



$$\int_1^4 \frac{1}{\sqrt{x}} dx = \int_1^4 x^{-\frac{1}{2}} dx = \left[2x^{\frac{1}{2}}\right]_1^4 = 2 \cdot 2 - 2 = 2 \text{ a. e.}$$

21. a)

$$N'(t) = 0.36e^{0.04t} \Rightarrow N(t) = 9e^{0.04t} + 3$$

b)

$$N(15) = 9e^{0.04 \cdot 15} + 3 \approx 19 \text{ miljoner}$$

22. a)

$$N'(t) = 120t \Rightarrow N(t) = 60t^2 + 1500$$

b)

$$N(3) = 60 \cdot 3^2 + 1500 = 2\,040 \text{ st}$$

23.

$$v(t) = 20 \cdot e^{-0.2t} \Rightarrow \int_0^{10} 20 \cdot e^{-0.2t} dt = 100[e^{-0.2t}]_{10}^{0} = 100(1 - e^{-2}) \approx 86 \text{ liter}$$

24. Den svarta kurvan: $y(x) = 3e^{kx}, y(2) = 2 = 3e^{2k} \Rightarrow k = \frac{1}{2}\ln\frac{2}{3} \Rightarrow y(x) = 3e^{x\frac{1}{2}\ln\frac{2}{3}}$

Den röda kurvan: $y(x) = e^{kx}, y(5) = 3 = e^{5x} \Rightarrow k = \frac{1}{5}\ln 3 \Rightarrow y(x) = e^{x\frac{1}{5}\ln 3}$

25. a)

$$y(x) = 1\,013 \cdot e^{-0.145x} \Rightarrow y(2) \approx 758 \text{ mbar}$$

b)

$$580 = 1013 \cdot e^{-0.145x} \Rightarrow x = 3.8 \text{ km}$$

c)

$$y'(x) = -0.145 \cdot 1\,013 \cdot e^{-0.145 \cdot 2} \text{ mbar/km}$$

d)

$$y'(2) = -0.145 \cdot 1\,013 \cdot e^{-0.145 \cdot 2} \approx -110 \text{ mbar/km}$$

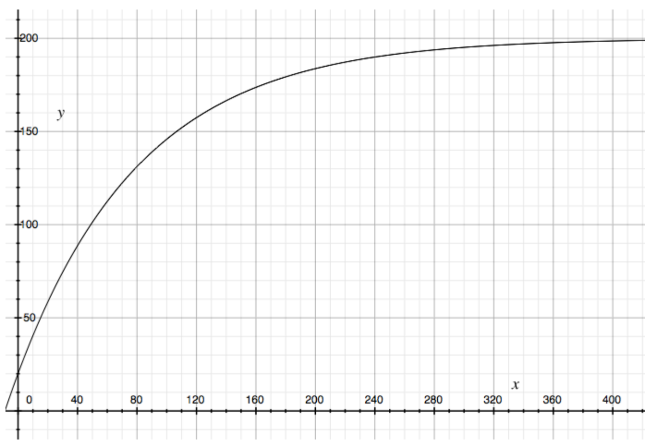
e)

$$y'(x) = -0.145 \cdot 1\,013 \cdot e^{-0.145 \cdot x} = -50 \text{ mbar/km} \Rightarrow x \approx 7.4 \text{ km höjd}$$

26.

$$y(t) = 200 - 180 \cdot e^{-kt}, y'(0) = 180k \cdot e^{-k0} = 2.08 \Rightarrow k = 0.012$$

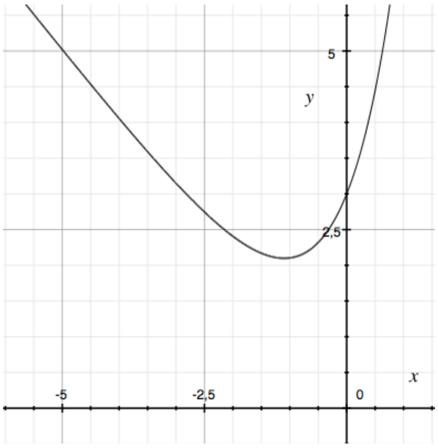
$$y(24) = 200 - 180 \cdot e^{-0.012 \cdot 24} \approx 64°\text{C}$$



27.

$$f(x) = 3e^x - x \Rightarrow f'(x) = 3e^x - 1 = 0 \Rightarrow x = \ln\frac{1}{3}$$

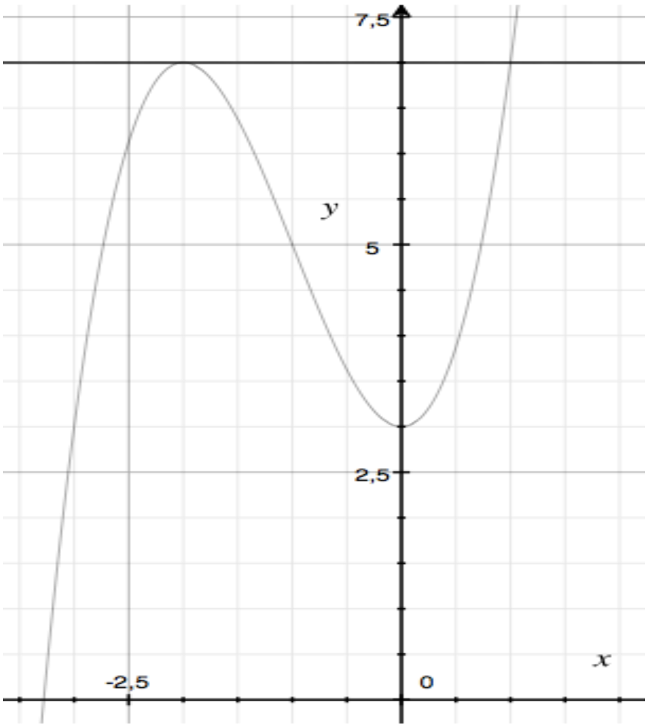
$$f\left(\ln\frac{1}{3}\right) \approx 2.1$$



28.

$$y(x) = x^3 + 3x^2 + 3 \Rightarrow y'(x) = 3x^2 + 6x = 0, x_1 = 0 \text{ och } x_2 = -2$$

$$\int_{-2}^{1} (7 - x^3 - 3x^2 - 3)dx = \left[4x - \frac{x^4}{4} - x^3\right]_{-2}^{1} = 3 - \frac{1}{4} - \left(-8 - \frac{16}{4} + 8\right) = 6\frac{3}{4} \text{ a. e.}$$



29. a)

$$B=\int_0^2 (2x-x^2)dx=\left[x^2-\frac{1}{3}x^3\right]_0^2=4-\frac{8}{3}=\frac{4}{3}\text{ a. e.}$$

b)

$$A+\frac{4}{3}=\frac{16}{3}\Rightarrow A=4\text{ a. e.}\Rightarrow\int_{-3}^{2} f(x)dx=B-A=\frac{4}{3}-4=-\frac{8}{3}$$

Blandade Uppgifter 4

39.

$$A=\int_0^1 xdx+\int_1^3\frac{1}{x^2}dx=\left[\frac{x^2}{2}\right]_0^1+\left[\frac{1}{x}\right]_3^1=\frac{1}{2}+1-\frac{1}{3}=\frac{7}{6}\text{ a. e.}$$

40. Om $f'(x)=0$ så är funktionen konstant dvs $f(x)=3\Rightarrow\int_2^6 3dx=12\text{ a. e.}$

41.

$$\int_0^1 x^n dx=\left[\frac{x^{n+1}}{n+1}\right]_0^1=\frac{1}{n+1}>\frac{1}{20}\Rightarrow 20>1+n\text{ dvs för }n<19.$$

47. a)

$$\int_{-3}^{2} f(x)dx=\int_{-3}^{-2} f(x)dx+\int_{-2}^{2} f(x)dx=-\frac{7}{6}+\frac{16}{3}=\frac{25}{6}\text{ a. e.}$$

b)

$$\int_{-3}^{3} f(x)+e^x dx=\int_{-3}^{3} f(x)dx+\int_{-3}^{3} e^x dx=-\frac{7}{6}+\frac{16}{3}-\frac{7}{6}+[e^x]_{-3}^{3}=$$

$$=-\frac{7}{6}+\frac{32}{6}-\frac{7}{6}+(e^3-e^{-3})=3+e^3-e^{-3}$$

48.

$$e^{2x}=p\Rightarrow x=\frac{1}{2}\ln p$$

$$\int_{0}^{\frac{1}{2}\ln p} p - e^{2x}dx = \left[px - \frac{1}{2}e^{2x}\right]_0^{\frac{1}{2}\ln p} = \frac{p}{2}\ln p - \frac{p}{2} + \frac{1}{2} = 10 \Rightarrow$$

$$p\ln p - p = 19 \Rightarrow p \approx 12.5$$

49. Kurvan skär x-axeln då $1 - kx^2 = 0$ dvs då $x = \pm k^{-\frac{1}{2}}$, alltså:

$$\int_{-1/\sqrt{k}}^{1/\sqrt{k}} 1 - kx^2dx = \left[x - k\frac{x^3}{3}\right]_{-1/\sqrt{k}}^{1/\sqrt{k}} = \left(\frac{1}{\sqrt{k}} - \frac{k}{3(\sqrt{k})^3}\right) - \left(-\frac{1}{\sqrt{k}} + \frac{k}{3(\sqrt{k})^3}\right) =$$

$$= \frac{1}{\sqrt{k}} - \frac{k}{3(\sqrt{k})^3} + \frac{1}{\sqrt{k}} - \frac{k}{3(\sqrt{k})^3} = \frac{2}{\sqrt{k}} - \frac{2}{3\sqrt{k}} = \frac{6}{3\sqrt{k}} - \frac{2}{3\sqrt{k}} = \frac{4}{3\sqrt{k}} = 2$$

$$\Rightarrow \sqrt{k} = \frac{2}{3} \Rightarrow k = \frac{4}{9}$$

50.

$$2a\int_1^a \frac{2 + x^3}{x^2}dx = 2a\int_1^a \frac{2}{x^2} + xdx = 2a\left[-\frac{2}{x} + \frac{x^2}{2}\right]_1^a = 2a\left[-\frac{2}{a} + \frac{a^2}{2} + \frac{2}{1} - \frac{1}{2}\right] =$$

$$= -\frac{4a}{a} + \frac{2a \cdot a^2}{2} + 4a - a = a^3 + 3a - 4 \text{ VSV}$$

51. $\ln(3x^2) - \ln x = 1 \Rightarrow \ln 3 + 2\ln x - \ln x = 1 \Rightarrow \ln x = 1 - \ln 3 = \ln e - \ln 3 = \ln\frac{e}{3}$

$$x = \frac{e}{3}$$