Valda uppgifter i kursboken Matematik M3c av Sjunnesson med flera utgiven på Liber, (2012).

Test 4

1. a)

$$f'(x) = 3e^x + 12e^{-3x}$$

b)

$$f(x) = 100 \cdot 1.03^x = 100 \cdot e^{x \ln 1.03} \Rightarrow f'(x) = 100 \cdot \ln 1.03 \cdot 1.03^x$$

2. a)

$$\ln x = 2 \Rightarrow x = e^2$$

b)

$$3e^x = 12 \Rightarrow e^x = 4 \Rightarrow x = \ln 4$$

3. a)

$$f(x) = 6x^2 - 4x^3 \Rightarrow F(x) = 2x^3 - x^4 + C$$

b)

$$f(x) = 2e^{x/3} \Rightarrow F(x) = 6e^{x/3} + C$$

4. a)

$$f(x) = \frac{3}{\sqrt{x}} \Rightarrow F(x) = 6\sqrt{x}$$

b)

$$g(x) = \frac{5}{x^2} \Rightarrow F(x) = -\frac{5}{x}$$

5. a)

$$f(x) = 3e^x - e^{-x/2} \Rightarrow F(x) = 3e^x + 2e^{-x/2} + C, F(0) = 3 + 2 + C = 1 \Rightarrow$$

$$F(x) = 3e^x + 2e^{-x/2} - 4$$

b)

$$f(x) = 3x^2 + 2x - 3 \Rightarrow F(x) = x^3 + x^2 - 3x + C, F(1) = 1 + 1 - 3 + C = 4 \Rightarrow$$

$$F(x) = x^3 + x^2 - 3x + 5$$

6.

$$v(t) = 5t \Rightarrow s(t) = 2.5t^2 + C, s(2) = 2.5 \cdot 2^2 + C = 6 \Rightarrow C = -4$$

$$s(t) = 2.5t^2 - 4 \text{ m}$$

7. a)

$$\int_0^3 4dx = 4[x]_0^3 = 4(3-0) = 12$$

b)

$$\int_2^6 (3t^2 - 2t + 1)dt = [t^3 - t^2 + t]_2^6 = 6^3 - 6^2 + 6 - (2^3 - 2^2 + 2) = 180$$

c)

$$\int_{-5}^0 2e^{-0.2x}dx = 10[e^{-0.2x}]_0^{-5} = 10(e-1)$$

10.

$$\int_1^3 (4x^7 - 2x^3 - 2x + 2)dx + \int_1^3 (-4x^7 + 2x^3 + 2x + 2)dx =$$

$$= \int_1^3 (4x^7 - 4x^7 - 2x^3 + 2x^3 - 2x + 2x + 2 + 2)dx =$$

$$= \int_1^3 4dx = 4[x]_1^3 = 4(3-1) = 8$$

11.

$$\int_0^{30} f(t)dt = 240 \text{ m}^3$$

Betyder att under 30 dagar förbrukades 240 m^3 vatten.

12.

$$g(t) = (e^t + 3)^2 = e^{2t} + 6e^t + 9 \Rightarrow G(t) = \frac{1}{2}e^{2t} + 6e^t + 9t + C \Rightarrow$$

$$G(0) = \frac{1}{2}e^0 + 6e^0 + 9 \cdot 0 + C = 3.5 \Rightarrow C = -3$$

$$G(t) = \frac{1}{2}e^{2t} + 6e^t + 9t - 3$$

13.

$$\int_1^a 2xdx = [x^2]_1^a = a^2 - 1 = 8 \Rightarrow a = 3$$

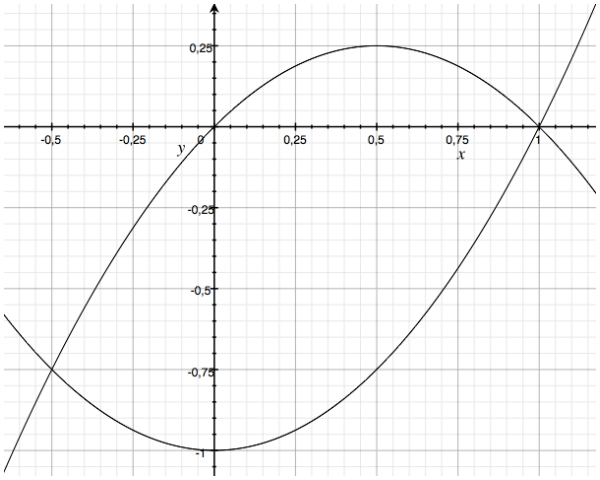
14.
a) ur grafen läses direkt att $F(2) = 5$

b) ur grafen fås direkt att $F(2) - F(0) = 5 - 1 = 4$

c)

$$\int_0^6 f(x)dx = [F(x)]_0^6 = F[6] - F[0] = 1 - 1 = 0$$

15.



$$\int_{-0.5}^{1} (x - x^2 - x^2 + 1)dx = \int_{-0.5}^{1} (x - 2x^2 + 1)dx = \left[\frac{1}{2}x^2 - \frac{2}{3}x^3 + x\right]_{-0.5}^{1} =$$

$$= \frac{1}{2}1^2 - \frac{2}{3}1^3 + 1 - \left(\frac{1}{2}(-0.5)^2 - \frac{2}{3}(-0.5)^3 - 0.5\right) =$$

$$= \frac{1}{2} - \frac{2}{3} + 1 - \left(\frac{1}{2}0.25 + \frac{2}{3}0.125 - 0.5\right) =$$

$$= \frac{12}{24} - \frac{16}{24} + \frac{24}{24} - \frac{3}{24} - \frac{2}{24} + \frac{12}{24} = \frac{27}{24} = \frac{9}{8} = 1.125$$

16. a)

$$\int e^x dx = e^x + C$$

$$\int_0^3 f(x)dx = e^3 - 1, \int_3^5 f(x)dx = e^5 - e^3 \text{ och } \int_0^5 f(x)dx = e^5 - 1$$

b)

$$\int_0^5 f(x)dx = \int_0^3 f(x)dx + \int_3^5 f(x)dx$$

17. a)

$$y = C \cdot a^x = 2000 \cdot 1.04^x$$

b)

$$y = C \cdot e^x = 2000 \cdot e^{x \ln 1.04}$$

c)

$$y'^{(5)} = 2000 \cdot \ln 1.04 \cdot e^{x \ln 1.04} \approx 95 \text{ kr/år}$$

Kapitaltillväxten efter 5 år är cirka 95 kr/år.

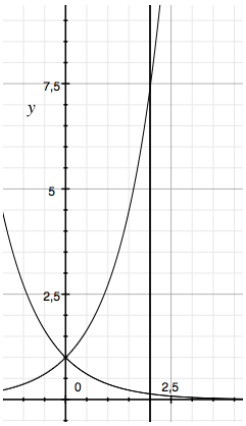
18. a)

$$\int_0^2 3x^2 dx = [x^3]_0^2 = 3^2 - 0^3 = 8 \text{ a. e.}$$

b)

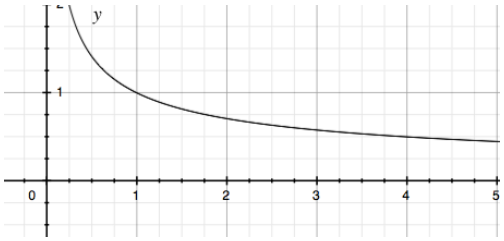
$$\int_0^1 0.5e^{2x} dx = [0.25e^{2x}]_0^1 = 0.25(e^2 - 1) \approx 1.6 \text{ a. e.}$$

19.



$$\int_0^2 (e^x - e^{-x}) dx = [e^x + e^{-x}]_0^2 = e^2 + e^{-2} - 2 \approx 5.5 \text{ a. e.}$$

20.



$$\int_1^4 \frac{1}{\sqrt{x}} dx = \int_1^4 x^{-\frac{1}{2}} dx = \left[2x^{\frac{1}{2}}\right]_1^4 = 2 \cdot 2 - 2 = 2 \text{ a. e.}$$

21. a)

$$N'(t) = 0.36e^{0.04t} \Rightarrow N(t) = 9e^{0.04t} + 3$$

b)

$$N(15) = 9e^{0.04 \cdot 15} + 3 \approx 19 \text{ miljoner}$$

22. a)

$$N'(t) = 120t \Rightarrow N(t) = 60t^2 + 1500$$

b)

$$N(3) = 60 \cdot 3^2 + 1500 = 2\,040 \text{ st}$$

23.

$$v(t) = 20 \cdot e^{-0.2t} \Rightarrow \int_0^{10} 20 \cdot e^{-0.2t} dt = 100[e^{-0.2t}]_{10}^0 = 100(1 - e^{-2}) \approx 86 \text{ liter}$$

24. Den svarta kurvan: $y(x) = 3e^{kx}, y(2) = 2 = 3e^{2k} \Rightarrow k = \frac{1}{2}\ln\frac{2}{3} \Rightarrow y(x) = 3e^{x\frac{1}{2}\ln\frac{2}{3}}$

Den röda kurvan: $y(x) = e^{kx}, y(5) = 3 = e^{5x} \Rightarrow k = \frac{1}{5}\ln 3 \Rightarrow y(x) = e^{x\frac{1}{5}\ln 3}$

25. a)

$$y(x) = 1\,013 \cdot e^{-0.145x} \Rightarrow y(2) \approx 758 \text{ mbar}$$

b)

$$580 = 1013 \cdot e^{-0.145x} \Rightarrow x = 3.8 \text{ km}$$

c)

$$y'(x) = -0.145 \cdot 1\,013 \cdot e^{-0.145 \cdot 2} \text{ mbar/km}$$

d)

$$y'(2) = -0.145 \cdot 1\,013 \cdot e^{-0.145 \cdot 2} \approx -110 \text{ mbar/km}$$

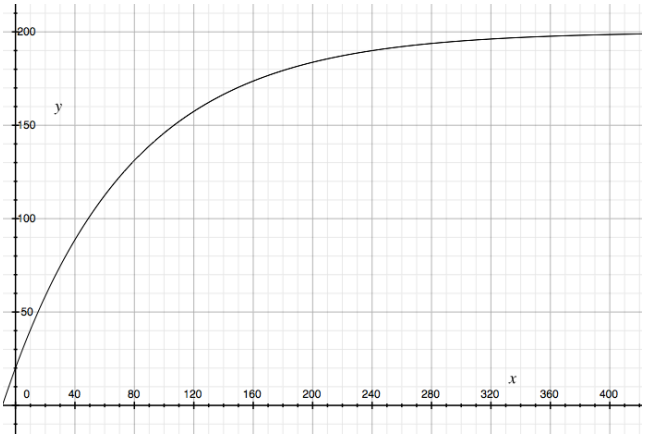
e)

$$y'(x) = -0.145 \cdot 1\,013 \cdot e^{-0.145 \cdot x} = -50 \text{ mbar/km} \Rightarrow x \approx 7.4 \text{ km höjd}$$

26.

$$y(t) = 200 - 180 \cdot e^{-kt}, y'(0) = 180k \cdot e^{-k0} = 2.08 \Rightarrow k = 0.012$$

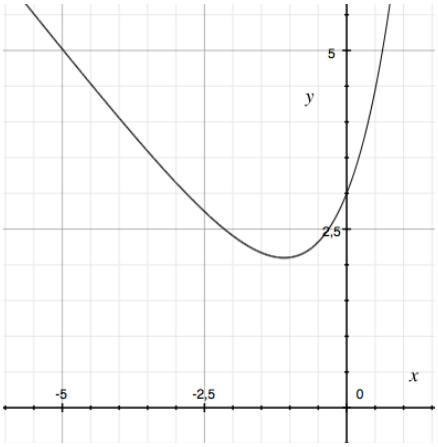
$$y(24) = 200 - 180 \cdot e^{-0.012 \cdot 24} \approx 64°\text{C}$$



27.

$$f(x) = 3e^x - x \Rightarrow f'(x) = 3e^x - 1 = 0 \Rightarrow x = \ln\frac{1}{3}$$

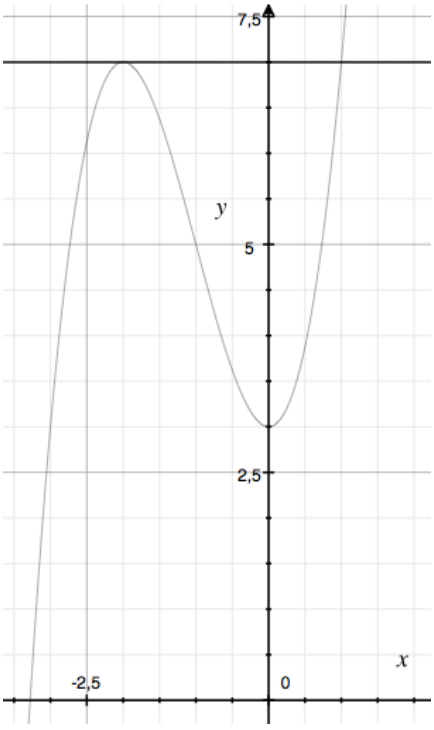
$$f\left(\ln\frac{1}{3}\right) \approx 2.1$$



28.

$$y(x) = x^3 + 3x^2 + 3 \Rightarrow y'(x) = 3x^2 + 6x = 0, x_1 = 0 \text{ och } x_2 = -2$$

$$\int_{-2}^{1}(7 - x^3 - 3x^2 - 3)dx = \left[4x - \frac{x^4}{4} - x^3\right]_{-2}^{1} = 3 - \frac{1}{4} - \left(-8 - \frac{16}{4} + 8\right) = 6\frac{3}{4} \text{ a. e.}$$



29. a)

$$B = \int_0^2 (2x - x^2)dx = \left[x^2 - \frac{1}{3}x^3\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3} \text{ a. e.}$$

b)

$$A + \frac{4}{3} = \frac{16}{3} \Rightarrow A = 4 \text{ a. e.} \Rightarrow \int_{-3}^{2} f(x)dx = B - A = \frac{4}{3} - 4 = -\frac{8}{3}$$