Valda uppgifter i kursboken Matematik M3c av Sjunnesson med flera utgiven på Liber, (2012).

2127. a)
$$\left(\sqrt{a} + \sqrt{b}\right)^2 - \left(\sqrt{a} - \sqrt{b}\right)^2 = a + 2\sqrt{ab} + b - \left(a - 2\sqrt{ab} + b\right) =$$

$$= a + 2\sqrt{ab} + b - a + 2\sqrt{ab} - b = 4\sqrt{ab}$$

b)

$$\left(1 + \sqrt{3}\right)^4 - 16\sqrt{3} = 1 + 4\sqrt{3} + 6 \cdot 3 + 4 \cdot 3\sqrt{3} + 9 - 16\sqrt{3} = 28$$

2128. Använd Pythagoras sats tre gånger och kombinera ihop.

$$d_1^2 = (a + x)^2 + h^2 \text{ och } d_2^2 = (a - x)^2 + h^2 \Rightarrow$$

$$d_1^2 + d_2^2 = a^2 + 2ax + x^2 + a^2 + 2ax + x^2 + 2h^2 = 2a^2 + 2x^2 + 2h^2$$

$$\text{men } b^2 = h^2 + x^2 \text{ dvs } d_1^2 + d_2^2 = 2a^2 + 2b^2 \text{ VSV}$$

2129.
$$h^4 + 9 = 10h^2 \Leftrightarrow h^4 - 10h^2 + 9 = 0 \Rightarrow h^2 = 5 \pm \sqrt{5^2 - 9} = 5 \pm 4 \Rightarrow$$

$$\Rightarrow \begin{cases} h_1^2 = 9 \\ h_2^2 = 1 \end{cases} = \begin{cases} h_1 = \pm 3 \\ h_2 = \pm 1 \end{cases}$$

2138. a)
$$1 - y^4 = (1 + y^2)(1 - y^2) = (1 + y^2)(1 + y)(1 - y)$$
b)
$$48t^5 - 3t = 3t(16t^4 - 1) = 3t(4t^2 + 1)(4t^2 - 1) = 3t(4t^2 + 1)(2t + 1)(2t - 1)$$
c)
$$x^{80} - 1 = (x^{40} + 1)(x^{40} - 1) = (x^{40} + 1)(x^{20} + 1)(x^{20} - 1) =$$

$$= (x^{40} + 1)(x^{20} + 1)(x^{10} + 1)(x^{10} - 1) =$$

$$= (x^{40} + 1)(x^{20} + 1)(x^{10} + 1)(x^5 + 1)(x^5 - 1)$$

2215.
$$f(x) = 3x + 16 \text{ och } g(x) = 2x - 3 \Rightarrow g(f(x)) = g(x) \text{ blir}$$

$$2(3x + 16) - 3 = 2x - 3 \Leftrightarrow 3x + 16 = x \Rightarrow x = -8$$

2216. Det är fel i uppgiften, det borde stått: $ax^n + bx^{n-1} + cx^{n-2}$

$$x^{3-n}(ax^n + bx^{n-1} + cx^{n-2}) = ax^3 + bx^2 + cx$$

2232.

$$(x-4)x\left(x+\frac{1}{2}\right) = 0$$

2233.

$$2x^2 - 12x - 14 = 2(x^2 - 6x - 7) = 0$$

Nollställen hittas som:

$$x = 3 \pm \sqrt{9+7} = 3 \pm 4 \Rightarrow$$

$$2x^2 - 12x - 14 = 2(x+1)(x-7)$$

2234.

$$x(x-3)^2 = 0$$

2235. a)

$$x^3 - 6x^2 + 10x = x(x^2 - 6x + 10)$$

b)

$$2x^2 - 16x + 14 = 2(x^2 - 8x + 7)$$

Nollställen hittas som:

$$x = 4 \pm \sqrt{16-7} = 4 \pm 3 \Rightarrow$$

$$= 2(x-7)(x-1)$$

c)

$$x^2 + 36x - 12x = x^2 + 24x = x(x+24)$$

d)

$$24 - 5x - x^2 = -(x^2 + 5x - 24)$$

Nollställen hittas som:

$$x = -2.5 \pm \sqrt{2.5^2 + 24} = -2.5 \pm 5.5 \Rightarrow$$

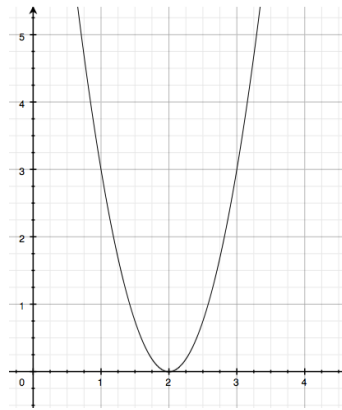$$-(x^2 + 5x - 24) = -(x+8)(x-3) = (x+8)(3-x)$$

2236. a)

$$4x = x^2 \Rightarrow x = 4 \text{ l. e. (eller } x = 0)$$

b)

$$6x^2 = x^3 \Rightarrow x = 6 \text{ l. e.}$$

2237. Ta till exempel $f(x) = 3(x - 2)^2$:

2238.
$$x^2 + bx + c = 0 \text{ dvs}(x - b)(x - c) = 0$$

$$x - x(b + c) + bc = 0 \Rightarrow \begin{cases} b = 1 \\ c = -2 \end{cases}$$

2316.
$$\frac{12}{x - 2}$$

Test 2

1. a) $2x - 6 = 2(x - 3)$ b) $6ab + 8b = 2b(3a + 4)$

c) $3x^4 - 6x^2 = 3x^2(x^2 - 2) = 3x^2\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)$

2.a) $1 - x^2 = (1 + x)(1 - x)$ b) $x^2 + 6x + 9 = (x + 3)^2$

c) $3 - 12x^2 = 3(1 - 4x^2) = 3(1 - 2x)(1 + 2x)$

3. a)
$$\frac{x \cdot x^{-4.2} \cdot x^{0.3}}{x^{-1.9}} = x^{1+0.3+1.9-4.2} = x^{-1}$$

b)
$$\frac{(a^5 b^{-1.5})^4}{a^4 \cdot b^{-6}} = \frac{a^{20} b^{-6}}{a^4 b^{-6}} = a^{16}$$

c)
$$\frac{(6a^2 b^3)^3}{(3a^{-2})^3 (4b^{0.5})^2} = \frac{6 \cdot 6 \cdot 6 \cdot a^6 b^9}{3 \cdot 3 \cdot 3 \cdot a^{-6} 4 \cdot 4 \cdot b} = \frac{a^{12} b^8}{2}$$

4.
$$x^4 + 3x^3 + 2x^2 + x + 1$$

5.
a) $x_1 = -1$ och $x_2 = 2$ b) $0 < x < 2$ c) $x < 2$

d) $x_1 = -1$ och $x_2 = 3$ e) $x = 2$ f) $x < -1$ och $x > 3$

g) $f(1) = -1$

6. a) $|-8| = 8$ b) $|3 - 5| = |-2| = 2$

7. a)
$$4(3x + 1)(3x - 1) - (4x + 1)(9x - 4) = 4(9x^2 - 1) - (36x^2 - 16x + 9x - 4) =$$

$$= 36x^2 - 4 - 36x^2 + 7x + 4 = 7x$$

b)
$$(3x - 2)^2 + (4x - 1)^2 - (5x - 2)^2 =$$

$$= 9x^2 - 12x + 4 + 16x^2 - 8x + 1 - (25x^2 - 20x + 4) =$$

$$= 25x^2 - 20x + 5 - 25x^2 + 20x - 4 = 1$$

8. a) $|2| - |7| = 2 - 7 = -5$ och $|2 - 7| = |-5| = 5$

b) $|-2| - |5| = 2 - 5 = -3$ och $|-2 - 5| = |-7| = 7$

9. a)
$$\frac{2 - x}{x^2 - 25} = \frac{2 - x}{(x + 5)(x - 5)} \text{ odefinierad för } x_1 = -5, x_2 = 5$$

b) Definierad för alla reella x.

10. a) $f(-2) = 2(-2)^2 - 3(-2) + 2 = 8 + 6 + 2 = 16$

b) $f(a) = 2a^2 - 3a + 2$

c) $f(a^2) = 2(a^2)^2 - 3a^2 + 2 = 2a^4 - 3a^2 + 2$

d) $f(a + h) - f(a) = 2(a + h)^2 - 3(a + h) + 2 - (2a^2 - 3a + 2) =$

$$= 2(a^2 + 2ah + h^2) - 3(a + h) + 2 - (2a^2 - 3a + 2) =$$

$$= 2a^2 + 4ah + 2h^2 - 3a - 3h + 2 - 2a^2 + 3a - 2 = 4ah + 2h^2 - 3h$$

11. a) $|x - 5| = 10 = \begin{cases} x_1 - 5 = 10 \\ x_2 - 5 = -10 \end{cases} = \begin{cases} x_1 = 15 \\ x_2 = -5 \end{cases}$

b) $|7 - x| = 4 = \begin{cases} 7 - x_1 = 4 \\ 7 - x_2 = -4 \end{cases} = \begin{cases} x_1 = 3 \\ x_2 = 11 \end{cases}$

12. a)
$$\frac{\dfrac{x^2 - x}{x + 1}}{x - 1} = \frac{x^2 - x}{x + 1} \cdot \frac{1}{x - 1} = \frac{x(x - 1)}{x + 1} \cdot \frac{1}{x - 1} = \frac{x}{x + 1}$$

b)

$$\frac{7}{10} + \frac{\frac{1}{2x}}{\frac{5}{3x}} = \frac{7}{10} + \frac{1}{2x}\frac{3x}{5} = \frac{7}{10} + \frac{1}{2}\frac{3}{5} = 1$$

13. a)
$$\frac{3}{x-1} - \frac{2}{x} = \frac{3x}{x(x-1)} - \frac{2(x-1)}{x(x-1)} = \frac{3x - 2(x-1)}{x(x-1)} = \frac{x+2}{x(x-1)}$$

b)
$$\frac{(x+3)^2}{x^2} - \left(1 + \frac{9}{x^2}\right) = \frac{(x+3)^2}{x^2} - \frac{x^2+9}{x^2} = \frac{x^2 + 6x + 9 - x^2 - 9}{x^2} = \frac{6}{x}$$

14. a)
$$\frac{1}{p^2-1} + \frac{1}{p+1} = \frac{1}{p^2-1} + \frac{p-1}{(p+1)(p-1)} = \frac{p}{p^2-1}$$

b)
$$\frac{1}{x+h} - \frac{1}{x} = \frac{x}{x(x+h)} - \frac{x+h}{x(x+h)} = -\frac{h}{x(x+h)}$$
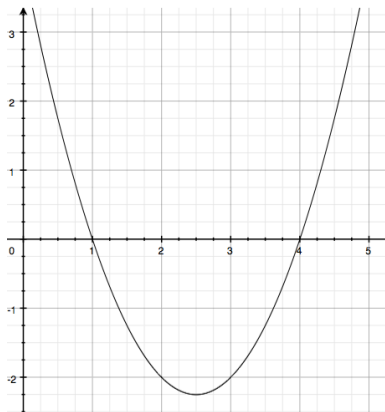
15. a)
$$x^2 + 8x + 15 = 0 \Rightarrow x = -4 \pm \sqrt{4^2 - 15} = -4 \pm 1 \Rightarrow$$
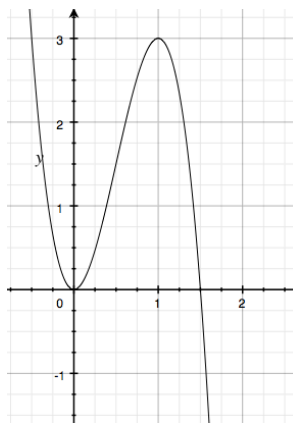
$$x^2 + 8x + 15 = (x+5)(x+3)$$

b)
$$2x^3 - 10x^2 + 12x = 2x(x^2 - 5x + 6) = 2x(x-3)(x-2)$$

16. a) Uttrycket byter tecken då $x = 4$ och då $x = 1$. För stora negativa x är uttrycket > 0. För stora positiva x är uttrycket > 0. Uttrycket är < 0 då $1 < x < 4$.

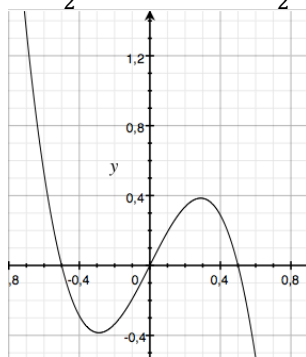b) $9x^2 - 6x^3 < 0 \Rightarrow 3x^2(3 - 2x) < 0$ byter tecken då $x = 1.5$, $3x^2(3 - 2x) < 0$ då $x > 1.5$.

17. a) $x^2 > 6x \Rightarrow x > 6$ eller $x < 0$

b) $2x - 8x^3 < 0 \Rightarrow 1 < 4x^2 \Rightarrow x < -\frac{1}{2}$ eller $0 < x < \frac{1}{2}$

18. a)

$$\frac{6}{2x} + \frac{3}{x} = \frac{3}{2} \Rightarrow 2x\left(\frac{6}{2x} + \frac{3}{x}\right) = 2x\frac{3}{2} \Rightarrow 6 + 6 = 3x \Rightarrow x = 4$$

b)

$$\frac{1}{2y-1} + \frac{1}{1+2y} - \frac{1}{y+2} = 0 \Rightarrow$$

$$\frac{(1+2y)(y+2) + (2y-1)(y+2) - (1+2y)(2y-1)}{(2y-1)(1+2y)(y+2)} = 0$$

$$(1+2y)(y+2) + (2y-1)(y+2) - (1+2y)(2y-1) = 0$$

$$y + 2 + 2y^2 + 4y + 2y^2 + 4y - y - 2 - (2y - 1 + 4y^2 - 2y) = 0$$

$$4y^2 + 8y - 2y + 1 - 4y^2 + 2y = 0$$

$$8y + 1 = 0 \Rightarrow y = -\frac{1}{8} = -0.125$$

19.

$$\frac{1}{b-1} + \frac{2}{b+1} - \frac{3}{b} = \frac{b(b+1) + 2b(b-1) - 3(b-1)(b+1)}{b(b-1)(b+1)} =$$

$$= \frac{b^2 + b + 2b^2 - 2b - 3(b^2 - 1)}{b(b^2 - 1)} = \frac{3-b}{b(b^2 - 1)}$$

20. a) och b)

$$\frac{\frac{h^2-h}{h+1}}{h^3-h} = \frac{h(h-1)}{h(h-1)(h+1)^2} = \left\{\begin{matrix} h \neq 0 \\ h \neq 1 \\ h \neq -1 \end{matrix}\right\} = \frac{1}{(h+1)^2} > 0 \text{ om } h \neq -1$$

21.

$$\frac{x^2+45x+200}{x^2-25} = x, x^2 + 45x + 200 = 0, x = -22.5 \pm 17.5 = \left\{\begin{matrix} -5 \\ -40 \end{matrix}\right.$$

$$\frac{(x+5)(x+40)}{(x+5)(x-5)} = x, x \neq -5 \Rightarrow x + 40 = x^2 - 5x$$

$$x^2 - 6x - 40 = 0 \Rightarrow x = 3 \pm 7 = \left\{\begin{matrix} x_1 = 10 \\ x_2 = -4 \end{matrix}\right.$$

Blandade uppgifter 2

1. När nämnaren $= 0$ är uttrycken inte definierade.

a) $x = -\frac{1}{2}$ b) $x = 3$ och $x = -2$

c) $x^2 - 9x = x(x-9)$ ej definierat då $x = 0$ och $x = 9$

2. a) $\frac{x}{3} + \frac{x}{2} = \frac{2x}{6} + \frac{3x}{6} = \frac{5x}{6}$

b) $\frac{1}{3a} - \frac{1}{4a} = \frac{4}{12a} - \frac{3}{12a} = \frac{1}{12a}$

c) $\frac{1}{x} - \frac{1}{2x} - \frac{1}{3x} = \frac{6}{6x} - \frac{3}{6x} - \frac{2}{6x} = \frac{1}{6x}$

3. a)

$$\frac{1}{x} - \frac{1}{x+1} = \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} = \frac{1}{x(x+1)}$$

b)

$$\frac{1}{x-1} + \frac{1}{x+1} = \frac{x+1}{(x-1)(x+1)} + \frac{x-1}{(x-1)(x+1)} = \frac{2x}{x^2-1}$$

c)

$$1 - \frac{1}{x+1} = \frac{x+1}{x+1} - \frac{1}{x+1} = \frac{x}{x+1}$$

4. a) $x^2 - 8x - 33 = 0 \Rightarrow x = 4 \pm \sqrt{4^2+33} = 4 \pm 7 = \begin{matrix} x_1 = 11 \\ x_2 = -3 \end{matrix}$

b) $3(x+5)(x-2) = 0$ behandla faktorerna var för sig $\Rightarrow \left\{\begin{matrix} x_1 = -5 \\ x_2 = 2 \end{matrix}\right.$

5. a) $|2x| = 8 \Rightarrow 2x = \pm 8 \Rightarrow x = \pm 4$

b) $|x + 9| = 2 \Rightarrow x + 9 = \pm 2 \Rightarrow \begin{cases} x_1 = -7 \\ x_2 = -11 \end{cases}$

6. a) $5x - 10 = 5(x - 2)$

b) $4x^2 - 2x = 2x(2x - 1)$

c) $3x^4 - 6x^3 + 12x^2 = 3x^2(x^2 - 2x + 4)$

7. a) $5 - 20x^2 = 5(1 - 4x^2)5(1 + 2x)(1 - 2x)$

b) $4x^2 + 28x + 49 = \{\text{lös andragradsekvation och hitta nollställen}\} = (2x + 7)^2$

eller titta i formelbladet på första kvadreringsregeln.

c) $x^3 - x^2 + \frac{x}{4} = x\left(x^2 - x + \frac{1}{4}\right) = x\left(x - \frac{1}{2}\right)^2$

8. $(1 - x)^4 = \left((-1)(x - 1)\right)^4 = (-1)^4(x - 1)^4 = (x - 1)^4$

9. a) $|x| - |y| + |x - y| = |7| - |-2| + |7 - (-2)| = 7 - 2 + 9 = 14$

b) $|x| - |y| + |x - y| = |-1| - |-5| + |-1 - (-5)| = 1 - 5 + 4 = 0$

10. a) $g(x)$ är mindre än 0 då x är mindre än -3. Svar: $x < -3$

b) $g(x) > 2$ när $x > 3$

c) $f(x) = 3$ när $x = 2$ dvs $f(2) = 3$ och när $x \approx -3.9$ dvs $f(-3.9) \approx 3$

d) Kurvorna skär varandra när $x = -3, \; x = 0$ och $x = 3$.

e) Kurvan $f(x)$ ligger ovanför kurvan $g(x)$ när $x < -3$ och när $0 < x < 3$.

11. a)
$$\frac{1}{x} + \frac{2}{5x} = \frac{5}{5x} + \frac{2}{5x} = \frac{7}{5x}$$

b)
$$\frac{2}{x} - \frac{1}{3x} = \frac{6}{3x} - \frac{1}{3x} = \frac{5}{3x}$$

c)
$$\frac{1}{2x} + \frac{1}{6x} = \frac{3}{6x} + \frac{1}{6x} = \frac{4}{6x} = \frac{2}{3x}$$

12 a)

$$\frac{5x}{y} \bigg/ \frac{x}{2y} = \frac{\frac{5x}{y}}{\frac{x}{2y}} = \frac{5x}{y} \cdot \frac{2y}{x} = 10$$

b)

$$\frac{4x^2}{y^2} \bigg/ \frac{4x}{3y} = \frac{\frac{4x^2}{y^2}}{\frac{4x}{3y}} = \frac{4x^2}{y^2} \cdot \frac{3y}{4x} = \frac{3x}{y}$$

c)

$$\frac{14yx}{5z^2} \bigg/ \frac{7y^2}{10z} = \frac{\frac{14yx}{5z^2}}{\frac{7y^2}{10z}} = \frac{14yx}{5z^2} \cdot \frac{10z}{7y^2} = \frac{4x}{yz}$$

13. a)

$$\frac{12x^3}{5y} \bigg/ \frac{8x^2}{3y^2} = \frac{\frac{12x^3}{5y}}{\frac{8x^2}{3y^2}} = \frac{12x^3}{5y} \cdot \frac{3y^2}{8x^2} = \frac{3x \cdot 3y}{5 \cdot 2} = \frac{9xy}{10}$$

b)

$$\frac{5x}{18y^2} \bigg/ \frac{15x}{6y^2} = \frac{\frac{5x}{18y^2}}{\frac{15x}{6y^2}} = \frac{5x}{18y^2} \cdot \frac{6y^2}{15x} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

c)

$$\frac{9a^2}{5b^2} \bigg/ \frac{9a^2}{6b^2} = \frac{\frac{9a^2}{5b^2}}{\frac{9a^2}{6b^2}} = \frac{9a^2}{5b^2} \cdot \frac{6b^2}{9a^2} = \frac{6}{5}$$

14. a) $v^2 = 2.4^2 + 2 \cdot 9.82 \cdot 12, v > 0 \Rightarrow v = \sqrt{2.4^2 + 2 \cdot 9.82 \cdot 12} \approx 15.5$

b)

$$15 = 2t + \frac{9.82t^2}{2}, t > 0 \Rightarrow t^2 + \frac{4}{9.82}t - \frac{30}{9.82} = 0 \Rightarrow t = -\frac{2}{9.82} + \sqrt{\left(\frac{2}{9.82}\right)^2 + \frac{30}{9.82}}$$

$$t \approx 1.56$$

15. $x^2 - 36 > 0 \Leftrightarrow (x - 6)(x + 6) > 0$ Gör ett teckenschema:

	−7	−6	0	6	7
$x - 6$	−	−	−	0	+
$x + 6$	−	0	+	+	+
produkt	+	0	−	0	+

Svar: $x^2 - 36 > 0$ när $x < -6$ eller $x > 6$

16. a)

$$\frac{1-a}{1-a^2} = \frac{1-a}{(1-a)(1+a)} = \frac{1}{1+a}$$

b)

$$\frac{x^2+4x+4}{x^2-4} = \frac{(x+2)^2}{(x+2)(x-2)} = \frac{x+2}{x-2}$$

c)

$$\frac{x^2-3x}{2x^2-12x+18} = \frac{x(x-3)}{2(x^2-6x+9)} = \frac{x(x-3)}{2(x-3)^2} = \frac{x}{2(x-3)}$$

17. a)

$$\frac{4x^2-1}{4x^2-4x+1} = \frac{(2x-1)(2x+1)}{(2x-1)^2} = \frac{2x+1}{2x-1}$$

b)

$$\frac{4x^2-9y^2}{4x^2-6xy} = \frac{(2x+3y)(2x-3y)}{2x(2x-3y)} = \frac{2x+3y}{2x}$$

c)

$$\frac{a+a^3}{a-a^5} = \frac{a(1+a^2)}{a(1+a^2)(1-a^2)} = \frac{1}{1-a^2}$$

18. a)

$$\left(\frac{2}{x}+\frac{3}{x+1}\right)\Big/\frac{10x+4}{x+1} = \frac{\frac{2}{x}+\frac{3}{x+1}}{\frac{10x+4}{x+1}} = \frac{\frac{2(x+1)+3x}{x(x+1)}}{\frac{10x+4}{x+1}} = \frac{5x+2}{x(10x+4)} =$$

$$= \frac{5x+2}{x2(5x+2)} = \frac{1}{2x}$$

b)

$$\left(\frac{1}{x-h}-\frac{1}{x}\right)\Big/\frac{1}{x-h} = \frac{\frac{x-(x-h)}{x(x-h)}}{\frac{1}{x-h}} = \frac{h}{x}$$

19. a)

$$G(300) = \frac{3000}{300} + 0.002x + 2.5 = 13.10 \text{ kr/st}$$

b)

$$G(500) - G(1000) = \frac{3000}{500} + 0.002 \cdot 500 - \frac{3000}{1000} - 0.002 \cdot 1000 = 2.00\frac{\text{kr}}{\text{st}} \text{ mindre}$$

Eller minskar från 9.50 kr/st till 7.50 kr/st.

20. a) $x^2 + 2x > 0 \Leftrightarrow x(x+2) > 0$ Gör ett teckenschema:

	−3	−2	−1	0	1
x	−	−	−	0	+
$x+2$	−	0	+	+	+
produkt	+	0	−	0	+

$$x^2 + 2x > 0 \text{ då } x < -2 \text{ eller } x > 0$$

b) $x^2 \leq x \Leftrightarrow x(x-1) \leq 0$ Gör ett teckenschema:

	−1	0	0.5	1	2
x	−	0	+	+	+
$x - 1$	−	−	−	0	+
produkt	+	0	−	0	+

Svar: $0 \leq x \leq 1$

21. a) $4x - 6x^2 < 0 \Leftrightarrow 4x(1 - 1.5x) < 0$ Gör ett teckenschema:

	−1	0	1/3	2/3	1
x	−	0	+	+	+
$1 - 1.5x$	+	+	+	0	−
produkt	−	0	+	0	−

Svar: $x < 0$ eller $x > 2/3$

b) $x^2 + x < 2 \Leftrightarrow x^2 + x - 2 < 0 \Leftrightarrow (x-1)(x+2) < 0$ Gör ett teckenschema:

	−3	−2	0	1	2
$x + 2$	−	0	+	+	+
$x - 1$	−	−	−	0	+
produkt	+	0	−	0	+

Svar: $-2 < x < 1$

22. a) $(2 - x)(3 + x) > 0$ Gör ett teckenschema:

	−4	−3	0	2	3
$2 - x$	+	+	+	0	−
$3 + x$	−	0	+	+	+
produkt	−	0	+	0	−

Svar: $-3 < x < 2$

b) $4x - x^3 < 0 \Leftrightarrow x(4 - x^2) = x(2 + x)(2 - x) < 0$ Gör ett teckenschema:

	−3	−2	−1	0	1	2	3
x	−	−	−	0	+	+	+
$2 - x$	+	+	+	+	+	0	−
$2 + x$	−	0	+	+	+	+	+
produkt	+	0	−	0	+	0	−

Svar: $-2 < x < -0$ eller $x > 2$

23.
$$\frac{x}{x^2 - h^2} - \frac{1}{x + h} = \frac{x}{(x+h)(x-h)} - \frac{x-h}{(x+h)(x-h)} = \frac{h}{(x+h)(x-h)}$$

24. Till exempel:
$$\frac{-6}{x - 3} = \frac{6}{3 - x}$$

25. a)
$$\frac{\dfrac{x^2 - 6x + 9}{x}}{\dfrac{x^2 - 9}{2x}} = \frac{(x-3)^2}{x} \cdot \frac{2x}{(x-3)(x+3)} = \frac{2(x-3)}{x+3}$$

b)
$$\frac{\dfrac{x^2 - 2xy}{x + y}}{\dfrac{x - 2y}{x^2 - y^2}} = \frac{x(x-2y)}{x+y} \cdot \frac{(x-y)(x+y)}{x-2y} = x(x-y)$$

26. a)

$$\frac{9-a^2}{2}/(a^2-6a+9) = \frac{(3-a)(3+a)}{2(a-3)^2} = \frac{3+a}{2(3-a)}$$

b)

$$\frac{\dfrac{a}{3}+\dfrac{a}{4}}{\dfrac{a}{3}-\dfrac{a}{4}} = \frac{\dfrac{4a}{12}+\dfrac{3a}{12}}{\dfrac{4a}{12}-\dfrac{3a}{12}} = \frac{7a}{a} = 7$$

27. a) $x^3 - 8x^2 = -7x \Rightarrow x(x^2-8x+7) = 0 \Rightarrow x(x-1)(x-7) = 0$

$$x_1 = 0, x_2 = 1 \text{ och } x_3 = 7$$

b) $4x^3 - 4x^2 + x = 0 \Rightarrow x(4x^2 - 4x + 1) = x(2x-1)^2 = 0$

$$x_1 = 0, x_2 = x_3 = \frac{1}{2}$$

c) $4x^2 - 12x^3 = 4x^2(1-3x) = 0 \Rightarrow x_1 = x_2 = 0, \ x_3 = \frac{1}{3}$

28. a)

$$8x - \frac{1}{2x} = 0 \Rightarrow 8x = \frac{1}{2x} \Rightarrow x^2 = \frac{1}{16} \Rightarrow x = \pm\frac{1}{4}$$

b)

$$\frac{y}{y-2} - \frac{3}{y} = 1 \Rightarrow y^2 - 3(y-2) = y^2 - 2y \Rightarrow y = 6$$

c)

$$\frac{1}{1-x} + \frac{1}{1+x} = 3 \Rightarrow 1 + x + 1 - x = 3 - 3x^2 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

29. a) $|x-8| = 5 \Rightarrow \begin{cases} x_1 - 8 = -5 \\ x_2 - 8 = 5 \end{cases} \Rightarrow \begin{cases} x_1 = 3 \\ x_2 = 13 \end{cases}$

b) $|2x-3| = 2 \Rightarrow \begin{cases} 2x_1 - 3 = -2 \\ 2x_2 - 3 = 2 \end{cases} \Rightarrow \begin{cases} x_1 = 1/2 \\ x_2 = 5/2 \end{cases}$

c) $|1-x| = x \Rightarrow \begin{cases} 1 - x_1 = x_1 \\ 1 - x_2 = -x_2 \end{cases} \Rightarrow \begin{cases} x_1 = 1/2 \\ \text{saknar lösning} \end{cases}$

30. a)

$$\frac{4}{x-2} - \frac{8}{x(x-2)} = \frac{4x-8}{x(x-2)} = \frac{4(x-2)}{x(x-2)} = \frac{4}{x}$$

Ej definierat då $x = 0$.

b) Se a).

c) $\frac{4}{x} = 2 \Rightarrow x = 2$ Svaret i facit är egendomligt.

31.
$$\frac{(8-2x)^4}{(4-x)^5} = \frac{2^4(4-x)^4}{(4-x)^5} = \frac{16}{4-x}$$

32.
$$p\big(p(-1)\big) = p\left(\frac{1+1}{-1}\right) = p(-2) = \frac{4+1}{-2} = -\frac{5}{2}$$

33.
$$H_B - H_A = \frac{2n(2n-1)}{2} - \frac{n(n-1)}{2} = \frac{4n^2 - 2n - n^2 + n}{2} = \frac{3n^2 - n}{2}$$

34. a) $12 - 4x - x^2 = \{\text{sök nollställen hos andragradsuttrycket}\} = (2-x)(6+x)$

b) $2x^3 + 16x - 18x^2 = 2x(x^2 - 9x + 8) = 2x(x-1)(x-8)$

c) $15x - 2x^2 - x^3 = -x(x^2 + 2x - 15) = -x(x-3)(x+5)$

35. a)
$$\frac{x^3 - 7x^2 - 18x}{2x^2 - 20x + 18} = \frac{x(x^2 - 7x - 18)}{2(x^2 - 10x + 9)} = \frac{x(x+2)(x-9)}{2(x-1)(x-9)} = \frac{x(x+2)}{2(x-1)}$$
b)
$$\frac{x^2 - 10x + 24}{x-4} + 6 = \frac{(x-4)(x-6)}{x-4} + 6 = x - 6 + 6 = x$$

36. a)
$$\frac{x^2 + x - 6}{x-2} + \frac{x^2 + 2x - 15}{x+5} = \frac{(x-2)(x+3)}{x-2} + \frac{(x-3)(x+5)}{x+5} = 2x$$
b)
$$\frac{x^2 - x - 12}{x^2 + 10x + 21} + \frac{4x-4}{x^2 + 6x - 7} - \frac{x}{x+7} = \frac{(x-4)(x+3)}{(x+3)(x+7)} + \frac{4(x-1)}{(x+7)(x-1)} - \frac{x}{x+7} =$$
$$= \frac{x-4}{x+7} + \frac{4}{x+7} - \frac{x}{x+7} = 0$$

37.
$$\frac{2}{x+1} + \frac{4}{x+2} = \frac{6}{x+3} \Leftrightarrow$$

$$\Leftrightarrow \frac{2(x+2)(x+3)}{(x+1)(x+2)(x+3)} + \frac{4(x+1)(x+3)}{(x+1)(x+2)(x+3)} = \frac{6(x+1)(x+2)}{(x+1)(x+2)(x+3)} \Leftrightarrow$$

$$\Leftrightarrow 2(x+2)(x+3) + 4(x+1)(x+3) = 6(x+1)(x+2) \Leftrightarrow$$

$$\Leftrightarrow x^2 + 5x + 6 + 2x^2 + 8x + 6 = 3x^2 + 9x + 6 \Leftrightarrow 4x = -6 \Leftrightarrow x = -\frac{3}{2}$$

38.

$$\begin{cases} 2a + 2b = 86 \\ ab = 420 \end{cases} \Rightarrow 2a + 2\frac{420}{a} = 86 \Rightarrow a^2 - 43a + 420 = 0 \Rightarrow$$

$$a = 21.5 \pm \sqrt{21.5^2 - 420} \Rightarrow \begin{aligned} a_1 &= 28 \text{ m} \\ a_2 &= 15 \text{ m} \end{aligned} \quad \text{svar: sidorna är 15 och 28 m}$$

39.

$$\frac{f(2+h) - f(2)}{h} = \frac{1}{h}\left(\frac{1}{2+h-1} - \frac{1}{2-1}\right) = \frac{1}{h}\left(\frac{1}{1+h} - 1\right) = \frac{1}{h}\left(\frac{1}{1+h} - \frac{1+h}{1+h}\right) =$$

$$= \frac{1}{h}\frac{-h}{(1+h)} = -\frac{1}{1+h}$$

40.

$$\left(x + \sqrt{x}\right)^2 - \left(x - \sqrt{x}\right)^2 = x^2 + 2x\sqrt{x} + x - x^2 + 2x\sqrt{x} - x = 4x\sqrt{x} \text{ VSV}$$

41.

$$x^2 + y^2 = \left(\frac{1-a^2}{1+a^2}\right)^2 + \left(\frac{2a}{1+a^2}\right)^2 = \frac{1 - 2a^2 + a^4}{(1+a^2)^2} + \frac{4a^2}{(1+a^2)^2} =$$

$$= \frac{1 + 2a^2 + a^4}{(1+a^2)^2} = \frac{(1+a^2)^2}{(1+a^2)^2} = 1 \text{ VSV}$$

42.

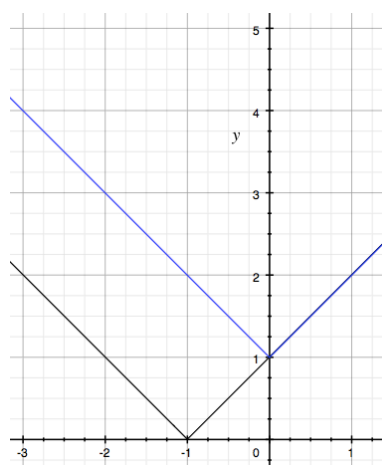$$p(x) = (2x - 1)^2, p\big(p(-2)\big) = x^4 \Rightarrow$$

$$p\big(p(-2)\big) = (2p(-2) - 1)^2 = (2(-2 \cdot 2 - 1)^2 - 1)^2 = x^4 \Rightarrow$$

$$(2(-2 \cdot 2 - 1)^2 - 1)^2 = x^4 \Rightarrow 49^2 = x^4 \Rightarrow x^2 = \pm 49 \Rightarrow x = \begin{cases} \pm 7 \\ \pm 7i \end{cases}$$

43.
$$|x + 1| < |x| + 1$$

I figuren ser man att uttrycket gäller för $x < 0$.