

Valda uppgifter i kursboken Matematik M3c av Sjunnesson med flera utgiven på Liber, (2012).

Test 2 ..... 1

## Test 2

1. a)  $2x - 6 = 2(x - 3)$    b)  $6ab + 8b = 2b(3a + 4)$

c)  $3x^4 - 6x^2 = 3x^2(x^2 - 2) = 3x^2(x - \sqrt{2})(x + \sqrt{2})$

2.a)  $1 - x^2 = (1 + x)(1 - x)$    b)  $x^2 + 6x + 9 = (x + 3)^2$

c)  $3 - 12x^2 = 3(1 - 4x^2) = 3(1 - 2x)(1 + 2x)$

3. a)

$$\frac{x \cdot x^{-4.2} \cdot x^{0.3}}{x^{-1.9}} = x^{1+0.3+1.9-4.2} = x^{-1}$$

b)

$$\frac{(a^5 b^{-1.5})^4}{a^4 \cdot b^{-6}} = \frac{a^{20} b^{-6}}{a^4 b^{-6}} = a^{16}$$

c)

$$\frac{(6a^2 b^3)^3}{(3a^{-2})^3 (4b^{0.5})^2} = \frac{6 \cdot 6 \cdot 6 \cdot a^6 b^9}{3 \cdot 3 \cdot 3 \cdot a^{-6} 4 \cdot 4 \cdot b} = \frac{a^{12} b^8}{2}$$

4.

$$x^4 + 3x^3 + 2x^2 + x + 1$$

5.

a)  $x_1 = -1$  och  $x_2 = 2$    b)  $0 < x < 2$    c)  $x < 2$

d)  $x_1 = -1$  och  $x_2 = 3$    e)  $x = 2$    f)  $x < -1$  och  $x > 3$

g)  $f(1) = -1$

6. a)  $|-8| = 8$    b)  $|3 - 5| = |-2| = 2$

7. a)

$$\begin{aligned} 4(3x + 1)(3x - 1) - (4x + 1)(9x - 4) &= 4(9x^2 - 1) - (36x^2 - 16x + 9x - 4) = \\ &= 36x^2 - 4 - 36x^2 + 7x + 4 = 7x \end{aligned}$$

b)

$$\begin{aligned} (3x - 2)^2 + (4x - 1)^2 - (5x - 2)^2 &= \\ = 9x^2 - 12x + 4 + 16x^2 - 8x + 1 - (25x^2 - 20x + 4) &= \end{aligned}$$

$$= 25x^2 - 20x + 5 - 25x^2 + 20x - 4 = 1$$

8. a)  $|2| - |7| = 2 - 7 = -5$  och  $|2 - 7| = |-5| = 5$

b)  $|-2| - |5| = 2 - 5 = -3$  och  $|-2 - 5| = |-7| = 7$

9. a)

$$\frac{2-x}{x^2-25} = \frac{2-x}{(x+5)(x-5)} \text{ odefinierad f\u00f6r } x_1 = -5, x_2 = 5$$

b) Definierad f\u00f6r alla reella  $x$ .

10. a)  $f(-2) = 2(-2)^2 - 3(-2) + 2 = 8 + 6 + 2 = 16$

b)  $f(a) = 2a^2 - 3a + 2$

c)  $f(a^2) = 2(a^2)^2 - 3a^2 + 2 = 2a^4 - 3a^2 + 2$

d)  $f(a+h) - f(a) = 2(a+h)^2 - 3(a+h) + 2 - (2a^2 - 3a + 2) =$

$$= 2(a^2 + 2ah + h^2) - 3(a+h) + 2 - (2a^2 - 3a + 2) =$$

$$= 2a^2 + 4ah + 2h^2 - 3a - 3h + 2 - 2a^2 + 3a - 2 = 4ah + 2h^2 - 3h$$

11. a)  $|x - 5| = 10 = \begin{cases} x_1 - 5 = 10 \\ x_2 - 5 = -10 \end{cases} = \begin{cases} x_1 = 15 \\ x_2 = -5 \end{cases}$

b)  $|7 - x| = 4 = \begin{cases} 7 - x_1 = 4 \\ 7 - x_2 = -4 \end{cases} = \begin{cases} x_1 = 3 \\ x_2 = 11 \end{cases}$

12. a)

$$\frac{\frac{x^2-x}{x+1}}{x-1} = \frac{x^2-x}{x+1} \cdot \frac{1}{x-1} = \frac{x(x-1)}{x+1} \cdot \frac{1}{x-1} = \frac{x}{x+1}$$

b)

$$\frac{7}{10} + \frac{\frac{1}{2x}}{\frac{5}{3x}} = \frac{7}{10} + \frac{1}{2x} \cdot \frac{3x}{5} = \frac{7}{10} + \frac{3}{10} = 1$$

13. a)

$$\frac{3}{x-1} - \frac{2}{x} = \frac{3x}{x(x-1)} - \frac{2(x-1)}{x(x-1)} = \frac{3x - 2(x-1)}{x(x-1)} = \frac{x+2}{x(x-1)}$$

b)

$$\frac{(x+3)^2}{x^2} - \left(1 + \frac{9}{x^2}\right) = \frac{(x+3)^2}{x^2} - \frac{x^2+9}{x^2} = \frac{x^2+6x+9-x^2-9}{x^2} = \frac{6}{x}$$

14. a)

$$\frac{1}{p^2-1} + \frac{1}{p+1} = \frac{1}{p^2-1} + \frac{p-1}{(p+1)(p-1)} = \frac{p}{p^2-1}$$

b)

$$\frac{1}{x+h} - \frac{1}{x} = \frac{x}{x(x+h)} - \frac{x+h}{x(x+h)} = -\frac{h}{x(x+h)}$$

15. a)

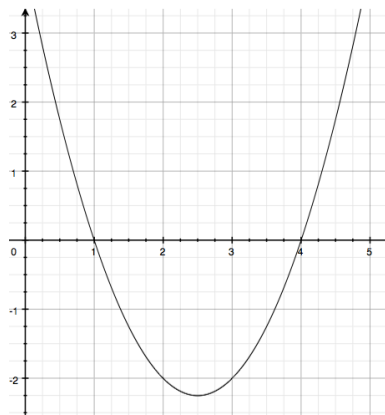
$$x^2 + 8x + 15 = 0 \Rightarrow x = -4 \pm \sqrt{4^2 - 15} = -4 \pm 1 \Rightarrow$$

$$x^2 + 8x + 15 = (x+5)(x+3)$$

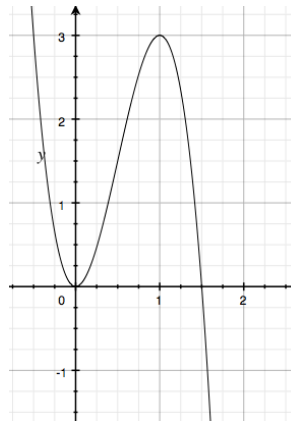
b)

$$2x^3 - 10x^2 + 12x = 2x(x^2 - 5x + 6) = 2x(x-3)(x-2)$$

16. a) Uttrycket byter tecken då  $x = 4$  och då  $x = 1$ . För stora negativa  $x$  är uttrycket  $> 0$ . För stora positiva  $x$  är uttrycket  $> 0$ . Uttrycket är  $< 0$  då  $1 < x < 4$ .

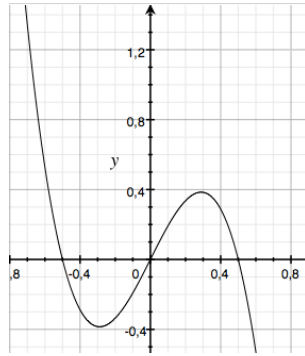


b)  $9x^2 - 6x^3 < 0 \Rightarrow 3x^2(3 - 2x) < 0$  byter tecken då  $x = 1.5$ ,  $3x^2(3 - 2x) < 0$  då  $x > 1.5$ .



17. a)  $x^2 > 6x \Rightarrow x > 6$  eller  $x < 0$

b)  $2x - 8x^3 < 0 \Rightarrow 1 < 4x^2 \Rightarrow x < -\frac{1}{2}$  eller  $0 < x < \frac{1}{2}$



18. a)

$$\frac{6}{2x} + \frac{3}{x} = \frac{3}{2} \Rightarrow 2x \left( \frac{6}{2x} + \frac{3}{x} \right) = 2x \frac{3}{2} \Rightarrow 6 + 6 = 3x \Rightarrow x = 4$$

b)

$$\frac{1}{2y-1} + \frac{1}{1+2y} - \frac{1}{y+2} = 0 \Rightarrow$$

$$\frac{(1+2y)(y+2) + (2y-1)(y+2) - (1+2y)(2y-1)}{(2y-1)(1+2y)(y+2)} = 0$$

$$(1+2y)(y+2) + (2y-1)(y+2) - (1+2y)(2y-1) = 0$$

$$y+2+2y^2+4y+2y^2+4y-y-2-(2y-1+4y^2-2y) = 0$$

$$4y^2+8y-2y+1-4y^2+2y = 0$$

$$8y+1 = 0 \Rightarrow y = -\frac{1}{8} = -0.125$$

19.

$$\frac{1}{b-1} + \frac{2}{b+1} - \frac{3}{b} = \frac{b(b+1) + 2b(b-1) - 3(b-1)(b+1)}{b(b-1)(b+1)} =$$

$$= \frac{b^2+b+2b^2-2b-3(b^2-1)}{b(b^2-1)} = \frac{3-b}{b(b^2-1)}$$

20. a) och b)

$$\frac{h^2-h}{h^3-h} = \frac{h(h-1)}{h(h-1)(h+1)^2} = \begin{cases} h \neq 0 \\ h \neq 1 \\ h \neq -1 \end{cases} = \frac{1}{(h+1)^2} > 0 \text{ om } h \neq -1$$

21.

$$\frac{x^2+45x+200}{x^2-25} = x, x^2+45x+200 = 0, x = -22.5 \pm 17.5 = \begin{cases} -5 \\ -40 \end{cases}$$

$$\frac{(x+5)(x+40)}{(x+5)(x-5)} = x, x \neq -5 \Rightarrow x+40 = x^2-5x$$

$$x^2-6x-40 = 0 \Rightarrow x = 3 \pm 7 = \begin{cases} x_1 = 10 \\ x_2 = -4 \end{cases}$$