Valda uppgifter i kursboken Matematik M2c av Sjunnesson med flera utgiven på Liber, (2011).

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Kapitel 1

1113. a) $1.4x - \frac{2}{5} \cdot (5 + 3.5x) = 1.4x - \left(2 + \frac{7}{5}x\right) = 1.4x - 2 - 1.4x = -2$

b) $1.5(2x - y) - (x - 0.6y) \cdot 3 = 3x - 1.5y - (3x - 1.8y) = 0.3y$

1114. a) $\frac{2}{3}(9 - 3x + 2y) - \left(\frac{y}{3} - \frac{x}{4} + 0.5\right) \cdot 4 = 6 - 2x + \frac{4}{3}y - \left(\frac{4}{3}y - x + 2\right) = 4 - x$

b) $4 - x = 4 - (-5) = 4 + 5 = 9$

1115. a) $C = A - B = 3 - 5x - (x + 3) = 3 - 5x - x - 3 = -6x$

b) $C = 2B - 3A = 2(x + 3) - 3(3 - 5x) = 2x + 6 - 9 + 15x = 17x - 3$

c) $C = 3B - Ax = 3(x + 3) - (3 - 5x)x = 3x + 9 - 3x + 5x^2 = 9 + 5x^2$

1116. a) $\frac{x}{2} - \frac{3x}{2} + \frac{5x}{2} - \frac{7x}{2} + \frac{9x}{2} = \frac{x-3x+5x-7x+9x}{2} = \frac{5x}{2}$
b) $\frac{5x}{2} = \frac{5 \cdot 16}{2} = 40$ c) $\frac{5x}{2} = \frac{5}{2} \cdot \left(-\frac{2}{15}\right) = -\frac{1}{3}$

1117. $\frac{7-2x}{3} - \frac{5-2x}{3} = \frac{7-2x-(5-2x)}{3} = \frac{7-2x-5+2x}{3} = \frac{2}{3}$

1118. a) $\bar{x} = \frac{1}{4}\left(2x + 9 + 13 + x + 3(x - 8) + 2(1 + x)\right) = \frac{1}{4}8x = 2x$
b) $\bar{x} = 2 \cdot 9.5 = 19$ c) $\bar{x} = -2 \cdot \frac{3}{8} = -\frac{3}{4}$

1119. a) $\sqrt{3}\left(\sqrt{12} + \sqrt{3}\right) = \left(\sqrt{3}\sqrt{12} + \sqrt{3}\sqrt{3}\right) = \sqrt{3 \cdot 12} + 3 = \sqrt{36} + 3 = 9$

b) $\sqrt{2}\left(\sqrt{8} - \sqrt{18}\right) = \left(\sqrt{2}\sqrt{8} - \sqrt{2}\sqrt{18}\right) = \sqrt{16} - \sqrt{36} = 4 - 6 = -2$

1120. $1 + x(2 - x) \cdot x - x^2(1 - x) = 1 + x^2(2 - x) - x^2(1 - x) =$

$$= 1 + (2x^2 - x^3) - (x^2 - x^3) = 1 + x^2 \geq 1 > 0$$

1135.

$$2(x + 4) + x = 50 \Rightarrow 3x = 42 \Rightarrow x = 14 \text{ cm}$$

1136.

$$(400 + 4x) \cdot 3 + 2(350 + 3x) = 2800 \Rightarrow$$

$$1200 + 12x + 700 + 6x = 2800 \Rightarrow 18x = 900 \Rightarrow x = 50$$

1137. a)

$$\begin{cases} B = 5000 + E \\ F = 3 \cdot E \\ B + F + E = 20\,000 \end{cases} \Rightarrow 5000 + E + 3E + E = 20\,000 \Rightarrow \begin{cases} E = 3000 \\ B = 8000 \\ F = 9000 \end{cases}$$

b) $\begin{cases} B = 5000 + E \\ F = 3 \cdot E \\ B + F + E = 40\,000 \end{cases} \Rightarrow 5000 + E + 3E + E = 40\,000 \Rightarrow \begin{cases} E = 7\,000 \\ B = 12\,000 \\ F = 21\,000 \end{cases}$

1138. a) $20 + \frac{x}{6} = x \Rightarrow 120 + x = 6x \Rightarrow x = \frac{120}{5} = 24$

b) $\frac{y}{3} + \frac{y}{2} = 15 \Rightarrow 6\frac{y}{3} + 6\frac{y}{2} = 6 \cdot 15 \Rightarrow 2y + 3y = 90 \Rightarrow y = \frac{90}{5} = 18$

1139. a) $\frac{3p}{2} + \frac{9}{10} = \frac{7p}{5}$ {förläng med 10} $\Rightarrow 5 \cdot 3p + 9 = 2 \cdot 7p \Rightarrow p = -9$

b) $\frac{1}{x} + \frac{2}{5} = \frac{3}{2x}$ {förläng med $10x$} $\Rightarrow 10 + 4x = 15 \Rightarrow x = \frac{5}{4}$

1140. a)

$$9 - \frac{3(4x - 2)}{5} = \frac{2 + x}{10}$$

förläng med 10:

$$90 - 6(4x - 2) = 2 + x \Rightarrow 90 - 24x + 12 = 2 + x \Rightarrow 100 = 25x \Rightarrow x = 4$$

b)

$$\frac{6y - 2}{5} - \frac{4y - 2}{3} = 1 - \frac{8 + y}{6}$$

förläng med 30:

$$30 \cdot \frac{6y - 2}{5} - 30 \cdot \frac{4y - 2}{3} = 30 \cdot \left(1 - \frac{8 + y}{6}\right)$$

$$6 \cdot (6y - 2) - 10 \cdot (4y - 2) = 30 - 5(8 + y)$$

$$36y - 12 - (40y - 20) = 30 - (40 + 5y)$$

$$36y - 12 - 40y + 20 = 30 - 40 - 5y$$

$$8 - 4y = -10 - 5y \Rightarrow y = -18$$

1141.

$$x - 12 = \frac{x}{3} \Rightarrow 3x - 36 = x \Rightarrow 2x = 36 \Rightarrow x = 18$$

1142. $\begin{cases} J = 2 \cdot E \\ M = J + 4 \\ E + J + M = 64 \end{cases} \Rightarrow \begin{cases} J = 2 \cdot E \\ M = 2E + 4 \\ E + 2E + 2E + 4 = 64 \end{cases} \Rightarrow 5E = 60 \Rightarrow M = 28$ år

1143. $\frac{x+2}{3x+6} = \frac{1}{3} \Rightarrow \frac{x+2}{3(x+2)} = \frac{1}{3}$ gäller för alla *x* utom $x = -2$.

1144. a) $\frac{1}{6} = \frac{2}{x-1} \Rightarrow x - 1 = 12 \Rightarrow x = 13$

b) $\frac{3}{x-2} = \frac{2}{x+1} \Rightarrow 3(x+1) = 2(x-2) \Rightarrow x = -7$

1145. $\frac{x}{25+x} = 0.02 \Rightarrow x = 0.5 + 0.02x \Rightarrow 0.98x = 0.5 \Rightarrow x = \frac{0.5}{0.98} \approx 5.1$ dl

1146. $\frac{1}{6} = \frac{2}{3-x^2} \Rightarrow 3 - x^2 = 12 \Rightarrow x^2 = -9$ dvs inga reella rötter.

1211.

$$3x(2x-5) - (x+8)(x-1) + 2(2.5x-1)(4-x) =$$

$$= 6x^2 - 15x - (x^2 - x + 8x - 8) + (5x-2)(4-x) =$$

$$= 6x^2 - 15x - x^2 + x - 8x + 8 + 20x - 5x^2 - 8 + 2x =$$

$$= -15x + x - 8x + 20x + 2x = 0$$

1213.

$$3 - (x-7)(2x+6) = 3 - (2x^2 + 6x - 14x - 42) =$$

$$= 3 - 2x^2 - 6x + 14x + 42 = 45 + 8x - 2x^2$$

1214.

$$(x+y+2)(3+x) - (x-y)(-3-x) = (x+y+2)(3+x) + (x-y)(3+x) =$$

$$= \big(x+y+2+(x-y)\big)(3+x) = (2+2x)(3+x) = 6 + 8x + 2x^2$$

1215. Kalla sidorna *x*, *y*, *u* och *z* analogt med problem 1212. Då gäller:

$$\begin{cases} xy = 117 \\ uy = 135 \\ xz = 156 \end{cases} \Rightarrow uz = \frac{135}{y} \cdot \frac{156}{x} = \frac{135 \cdot 156}{117} = 180 \text{ m}^2$$

1225. $(a-b) \cdot (a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$

1226. $(a-b)^2 = a^2 - 2ab + b^2 = b^2 - 2ab + a^2 = (b-a)^2$

1227. a) $\left(\sqrt{2} + \sqrt{8}\right)^2 = \left(\sqrt{2}\right)^2 + 2\sqrt{2}\sqrt{8} + \left(\sqrt{8}\right)^2 = 2 + 2\sqrt{16} + 8 = 18$

b) $\left(\sqrt{12} - \sqrt{3}\right)^2 = \left(\sqrt{12}\right)^2 - 2\sqrt{12}\sqrt{3} + \left(\sqrt{3}\right)^2 = 12 - 2\sqrt{36} + 3 = 3$

1228. a)

$$(x+h)^3-(x-h)^3=x^3+3x^2h+3xh^2+h^3-(x^3-3x^2h+3xh^2-h^3)=$$

$$=x^3+3x^2h+3xh^2+h^3-x^3+3x^2h-3xh^2+h^3=6x^2h+2h^3$$

b)

$$(x+2)^3-(x+2)^2-(x-2)=$$

$$=x^3+3x^22+3x2^2+2^3-(x^2+4x+4)-(x-2)=$$

$$=x^3+6x^2+12x+8-x^2-4x-4-x+2=$$

$$=x^3+5x^2+7x+6$$

1229 a) $\left(\sqrt{2x}+\sqrt{4.5x}\right)^2=2x+2\sqrt{2x}\sqrt{4.5x}+4.5x=6.5x+2\cdot 3x=12.5x$

b) $\left(\sqrt{20x}-x\sqrt{5x}\right)^2=20x-2\sqrt{20x}\cdot x\sqrt{5x}+x^25x=20x-2x10x+x^25x=$
$=20x-20x^2+5x^3$

c) $(5^{3x}-5^{0.5x})^2=5^{6x}-2\cdot 5^{3x}5^{0.5x}+5^x=5^{6x}-2\cdot 5^{3.5x}+5^x$

d) $\left(\frac{a+1}{2}\right)^2-\left(\frac{a-1}{2}\right)^2=\frac{a^2+2a+1}{4}-\frac{a^2-2a+1}{4}=\frac{a^2+2a+1-a^2+2a-1}{4}=a$

1230. Hela den stora kvadraten = 4 gula plus den röda. Eller matematiskt uttryckt:

$$c^2=4\frac{ab}{2}+(a-b)^2=2ab+a^2-2ab+b^2=a^2+b^2$$

1243.

$$\begin{cases}a-b=5\\a^2-b^2=195\end{cases}\Rightarrow\begin{cases}a-b=5\\(a+b)(a-b)=195\end{cases}\Rightarrow\begin{cases}a-b=5\\(a+b)5=195\end{cases}\Rightarrow\begin{cases}a-b=5\\a+b=39\end{cases}$$

$$\Rightarrow\begin{cases}a=22\\b=17\end{cases}$$

1244. a) $18^2=20\cdot 16+4=320+4=324,\ 14^2=10\cdot 18+16=196$
$39^2=40\cdot 38+1=1520+1=1521,\qquad 97^2=100\cdot 94+9=9409$

b) $a^2=(a-b)(a+b)+b^2=a^2-b^2+b^2=a^2$ VSV

1254.

$$(x+3)^2+8(3-x)^2=43-(5+3x)(5-3x)$$

$$(x^2+6x+9)+8(9-6x+x^2)=43-(25-9x^2)$$

$$x^2+6x+9+72-48x+8x^2=43-25+9x^2$$

$$9x^2+81-42x=18+9x^2\Rightarrow 63=42x\Rightarrow x=\frac{63}{42}=\frac{3}{2}=1.5$$

1255. $(3x+4)^2-(4-3x)^2=(2x+3)^2-(3-2x)^2$

$$(9x^2+24x+16)-(16-24x+9x^2)=(4x^2+12x+9)-(4x^2-12x+9)$$

$$48x=24x\Rightarrow x=0$$

1256. $x^2+x+19=(x+3)^2=x^2+6x+9\Rightarrow 5x=10\Rightarrow x=2$

1257. $5^2+x^2=(10-x)^2=100-20x+x^2\Rightarrow 20x=75\Rightarrow x=\frac{15}{4}=3.75$ cm

1258. a) $14^2+(35-x)^2=(35+x)^2\Rightarrow 196-70x=70x\Rightarrow x=\frac{196}{140}=\frac{14\cdot 14}{10\cdot 14}=1.4$ cm
Svar: Konens höjd är 33.6 cm

b) $V_{kon}=\frac{bh}{3}=\frac{3.36}{3}\pi 1.4^2\approx 7$ liter

1312. Fall 1: 15 är en katet och 4*x* är hypotenusa.

$15^2+4x^2=16x^2\Rightarrow 12x^2=225\Rightarrow x=\sqrt{\frac{225}{12}}\approx 4.3$ cm

Svar: 2*x*= 8.7 cm

Fall 2: 4*x* är en katet och 15 är hypotenusa.

$15^2=4x^2+16x^2\Rightarrow 20x^2=225\Rightarrow x=\sqrt{\frac{225}{20}}\approx 3.4$ cm

Svar: 2*x*= 6.7 cm

1313. a) $\frac{x}{3}=\frac{27}{x}\Rightarrow x^2=81\Rightarrow x=\pm 9$

b) $3x=\left(\sqrt{x}+3\right)\left(\sqrt{x}-3\right)+10=x-9+10=x+1\Rightarrow 2x=1\Rightarrow x=0.5$

1314. $\frac{3x-2}{x-1}=\frac{x-1}{x}\Rightarrow (3x-2)x=(x-1)^2\Rightarrow$
$3x^2-2x=x^2-2x+1\Rightarrow 2x^2=1\Rightarrow x^2=\frac{1}{2}\Rightarrow x=\pm\frac{1}{\sqrt{2}}=\pm\sqrt{0.5}$

1315. $(x+a)^2=(2a-3x)^2\Rightarrow\begin{cases}x_1+a=2a-3x_1\\x_2+a=-2a+3x_2\end{cases}\Rightarrow\begin{cases}4x_1=a\\2x_2=3a\end{cases}$

$$\Rightarrow\begin{cases}x_1=\frac{a}{4}=0.25a\\x_2=\frac{3a}{2}=1.5a\end{cases}$$

1316. $(6-a)^2=16\Rightarrow 6-a=\pm 4\Rightarrow a=6\mp 4\Rightarrow\begin{cases}a_1=2\\a_2=10\end{cases}\Rightarrow$

$$\begin{cases}(x-2)^2=16\\(x-10)^2=16\end{cases}\Rightarrow\begin{cases}x_1=6,x_2=-2\\x_3=14,x_4=6\end{cases}$$

1322.

$$(x+6)^2+x^2=20^2\Rightarrow x^2+12x+36+x^2=400\Rightarrow 2x^2+12x-364=0\Rightarrow$$

$$x^2+6x-182=0\Rightarrow x=-3\pm\sqrt{9+182}\approx -3\pm 13.8=10.8\text{ cm}$$

$$A_{\triangle}=\frac{10.8(10.8+6)}{2}\approx 91\text{ cm}^2$$

1323. $y^4-10y^2+9=0\Rightarrow y^2=5\pm\sqrt{25-9}=5\pm 4=\begin{cases}y_1^2=9\\y_2^2=1\end{cases}\Rightarrow\begin{cases}y_1=\pm 3\\y_2=\pm 1\end{cases}$

1338. $(x+1)(x-3)=0\Rightarrow x^2-3x+x-3=0\Rightarrow x^2-2x-3=0$

1339. $1.2=0.0001x^2-0.016x+1.34\Rightarrow 0.0001x^2-0.016x+0.14=0\Rightarrow$

$$x^2-160x+1400=0\Rightarrow x=80\pm\sqrt{80^2-1400}\approx 150\text{ km/h}$$

1340. a) $x^4-13x^2+36=0\Rightarrow x^2=6.5\pm\sqrt{6.5^2-36}=6.5\pm 2.5=\begin{cases}9\\4\end{cases}\Rightarrow$
$x^2=\begin{cases}9\\4\end{cases}\Rightarrow x=\begin{cases}\pm 3\\\pm 2\end{cases}$
b) $x-5\sqrt{x}+4=0\Rightarrow\sqrt{x}=2.5\pm\sqrt{2.5^2-4}=\begin{cases}4\\1\end{cases}\Rightarrow x=\begin{cases}16\\1\end{cases}$

1341. a) $ax^2+bx+c=0\Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0\Rightarrow p=\frac{b}{a}$ och $q=\frac{c}{a}$
b) $x=-\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q}=-\frac{b}{2a}\pm\sqrt{\left(\frac{b}{2a}\right)^2-\frac{c}{a}}=-\frac{b}{2a}\pm\sqrt{\frac{b^2}{4a^2}-\frac{4ac}{4a^2}}$

$$x=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\text{ VSB}$$

1352. a) $a=5\cdot 6, b=-11\Rightarrow x^2-11x+30=0$

b) $\begin{cases}r_1=-\frac{p}{2}+\sqrt{\frac{p^2}{4}-q}\\r_2=-\frac{p}{2}-\sqrt{\frac{p^2}{4}-q}\end{cases}\Rightarrow\begin{cases}r_1+r_2=-\frac{p}{2}+\sqrt{\frac{p^2}{4}-q}-\frac{p}{2}-\sqrt{\frac{p^2}{4}-q}\\r_1r_2=\left(-\frac{p}{2}+\sqrt{\frac{p^2}{4}-q}\right)\left(-\frac{p}{2}-\sqrt{\frac{p^2}{4}-q}\right)\end{cases}$

$$\begin{cases}r_1+r_2=-p\\r_1r_2=\frac{p^2}{4}-\left(\frac{p^2}{4}-q\right)=q\end{cases}$$

1362. a)

n	1	2	3	4	5	6	7	8	9	10	11	12
i^n	i	-1	$-i$	1	i	-1	$-i$	1	i	-1	$-i$	1

b) $i^n = 1$ när n är delbart med 4.
c) $i^{87} + i^{4001} = i^{84+3} + i^{4000+1} = i^3 + i^1 = -i + i = 0$

1363.
Lösning 1:
$2z^2 + 2(4-2i)z - 16i = 0 \Rightarrow z^2 + (4-2i)z - 8i = 0$

$z^2 + 4z - 2iz - 8i = 0 \Rightarrow z(z+4) - 2i(z+4) = (z+4)(z-2i) = 0$

Lösning 2: (med pq-formel, men mera tidskrävande)

$$z = -\frac{4-2i}{2} \pm \sqrt{\left(\frac{4-2i}{2}\right)^2 + 8i} = i - 2 \pm \sqrt{\frac{16-16i-4}{4} + 8i} =$$

$$= i - 2 \pm \sqrt{3 - 4i + 8i} = i - 2 \pm \sqrt{3+4i} = i - 2 \pm (2+i) = \begin{cases} 2i \\ -4 \end{cases}$$

1364. $z_1 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}$, $z_2 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}$, $z_1 + z_2 = -p = 12$,
$z_1 = a + bi$ och $z_2 = a - bi \Rightarrow z_1 + z_2 = 2a = 12 \Rightarrow a = 6$

Då rötterna är komplexa måste:

$$\frac{p^2}{4} - q < 0 \Rightarrow \frac{12^2}{4} - q < 0 \Rightarrow 36 - q < 0 \Rightarrow q > 36$$

1370. a) $\sqrt{3-x} = -x - 9 \Rightarrow 3 - x = x^2 + 18x + 81$

$$\Rightarrow x^2 + 19x + 78 = 0 \Rightarrow x = -9.5 \pm \sqrt{9.5^2 - 78} = -9.5 \pm 3.5 = \begin{cases} x_1 = -6 \\ x_2 = -13 \end{cases}$$

Test: $\begin{cases} \sqrt{3-(-6)} = -(-6) - 9 \\ \sqrt{3-(-13)} = -(-13) - 9 \end{cases} \Rightarrow \begin{cases} \sqrt{9} = -3 \text{ NEJ, falsk rot!} \\ \sqrt{16} = 4 \text{ Stämmer!} \end{cases}$

b) $\sqrt{x^2 + \sqrt{20x+70}} = x + 2 \Rightarrow x^2 + \sqrt{20x+70} = x^2 + 4x + 4 \Rightarrow$

$$\Rightarrow \sqrt{20x+70} = 4x + 4 \Rightarrow 20x + 70 = 16x^2 + 32x + 16 \Rightarrow$$

$$\Rightarrow 16x^2 + 12x - 54 = 0 \Rightarrow x^2 + \frac{3}{4}x - \frac{27}{8} = 0 \Rightarrow x = -\frac{3}{8} \pm \sqrt{\left(\frac{3}{8}\right)^2 + \frac{27}{8}}$$

$$\Rightarrow x = -\frac{3}{8} \pm \sqrt{\frac{225}{64}} = \frac{-3 \pm 15}{8} = \begin{cases} x_1 = \frac{3}{2} \text{ Riktig rot!} \\ x_2 = -\frac{9}{4} \text{falsk rot.} \end{cases}$$

1371. A. $\sqrt{2x+5} = x+1 \Rightarrow 2x+5 = x^2+2x+1 \Rightarrow x = \pm 2$ men -2 är en falsk rot.

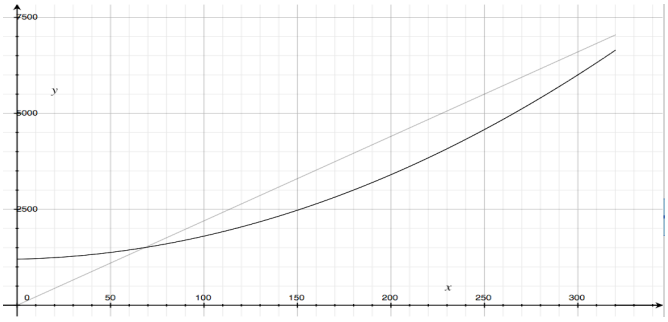
B. $\sqrt{2x+5} = -(x+1) \Rightarrow$ Bara $x = -2$ är lösning.

C. $2x+5 = (x+1)^2 \Rightarrow x = \pm 2$ båda är riktiga rötter.

1381.
a) $6000 = 1200 + x + 0.05x^2 \Rightarrow x^2 + 20x - 96000 = 0 \Rightarrow$

$x = -10 \pm \sqrt{100 + 96000} = 300$ st (den negativa roten stryks)

b) $22x = 1200 + x + 0.05x^2 \Rightarrow x^2 - 420x + 24000 = 0$
x
$= 210 \pm \sqrt{210^2 - 24000} = \begin{cases} 68 \text{ flaskor} \\ 352 \quad \text{(utanför definitionsmängden)} \end{cases}$



1382.
$\begin{cases} a+b = 51 \\ ab = 144 \end{cases} \Rightarrow a + \frac{144}{a} = 51 \Rightarrow a^2 - 51a + 144 = 0 \Rightarrow a = 25.5 \pm \sqrt{25.5^2 - 144}$

$$a = 25.5 \pm 22.5 = \begin{cases} 48 \\ 3 \end{cases} \Rightarrow \begin{cases} a_1 = 48 \\ b_1 = 3 \end{cases} \text{eller} \begin{cases} a_2 = 3 \\ b_2 = 48 \end{cases}$$

1383. $h = r + 16, 105 = 2\pi r^2 + 2\pi r(r+16) = 2\pi r^2 + 2\pi r^2 + 2\pi r 16 \Rightarrow$

$$4\pi r^2 + 2\pi r 16 - 105 = 0 \Rightarrow r^2 + \frac{2\pi 16}{4\pi} r - \frac{105}{4\pi} = 0 \Rightarrow r^2 + 8r - \frac{105}{4\pi} = 0 \Rightarrow$$

$$r = -4 \pm \sqrt{16 + \frac{105}{4\pi}} = 9.4 \text{ mm}$$

1384. Den lilla delen, 1-x, förhåller sig till den stora som den stora till det hela.

$\frac{1-x}{x} = \frac{x}{1} \Rightarrow 1 - x = x^2 \Rightarrow x^2 + x - 1 = 0 \Rightarrow x = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 1}$

$\Rightarrow x = -\frac{1}{2} \pm \sqrt{\frac{1+4}{4}} = \{\text{ignorera den negativa lösningen}\} = \frac{\sqrt{5}-1}{2}$

1412. a) $x^3 - 81x = x(x^2 - 81) = x(x+9)(x-9)$

b) $4(x+2)(x+7) - 8(x+2)(2x+3) = 4(x+2)\big((x+7) - 2(2x+3)\big) =$

$$= 4(x+2)(x+7-4x-6) = 4(x+2)(1-3x)$$

c) $(x+3)^2 - 9y^2 = (x+3-3y)(x+3+3y)$

d) $b^{3n} - b^n = b^n(b^{2n} - 1) = b^n(b^n + 1)(b^n - 1)$

1424. $x^2 + a = 0 \Rightarrow (x+7)(x-7) = x^2 - 49 \Rightarrow a = -49$

1425. $x^{80} - 1 = (x^{40} + 1)(x^{40} - 1) = (x^{40} + 1)(x^{20} + 1)(x^{20} - 1) =$

$= (x^{40} + 1)(x^{20} + 1)(x^{20} - 1) = (x^{40} + 1)(x^{20} + 1)(x^{10} + 1)(x^{10} - 1) =$

$= (x^{40} + 1)(x^{20} + 1)(x^{10} + 1)(x^5 + 1)(x^5 - 1)$

1439. $z^5 - za^4 = z(z^4 - a^4) = z(z^2 + a^2)(z^2 - a^2) = z(z^2 + a^2)(z + a)(z - a)$

$$= z(z^2 + 49)(z+7)(z-7) \Rightarrow z = 0, z = \pm 7 \text{ och } z = \pm 7i$$

1440. $f(3) = 3a(3+a) = 12 \Rightarrow a^2 + 3a - 4 = 0 \Rightarrow a = -1.5 \pm \sqrt{2.25 + 4} = \begin{cases} -4 \\ 1 \end{cases}$

$$f(x) = \begin{cases} -4x(x-4) = 0 \Rightarrow x_1 = 0, x_2 = 4 \\ x(x+1) = 0 \Rightarrow x_3 = 0, x_4 = -1 \end{cases}$$

Test 1

1. a) $(5-y)^2 = 25 - 10y + y^2$

b) $(1-y)^2 = 1 - 2y + y^2$

c) $(3y-2)^2 = 9y^2 - 12y + 4$

2. $(4x-3)(4x+3) + (x-9)^2 - 9(8-2x) =$

$$= 16x^2 - 9 + x^2 - 18x + 81 - 72 + 18x = 17x^2$$

3.
$(x-4)^2 + (x+1)(x+10) - 2x(1+x) = x^2 - 8x + 16 + x^2 + 11x + 10 - 2x - 2x^2 =$

$$= x + 26$$

15. a) $(x-5)3x = 0 \Rightarrow x_1 = 5$ och $x_2 = 0$

b) $(x-2)(x+7) = 0 \Rightarrow x_1 = 2$ och $x_2 = -7$

16. a) $4(8-4x)(2x+9)(3x-1) = 0$ nollställena hittas ett i varje faktor dvs

$$x_1 = 2, x_2 = -4.5 \text{ och } x_3 = 1/3$$

b) $x(2x+7)(5x-2) = 0$, samma metod som i a) dvs

$$x_1 = 0, x_2 = -3.5 \text{ och } x_3 = 0.4$$

17. a) $x^4 - 16x^2 = 0 \Leftrightarrow x^2(x^2-16) = x^2(x+4)(x-4) = 0$

$$\begin{cases} x_1 = x_2 = 0 \\ x_3 = 4 \\ x_4 = -4 \end{cases}$$

b) $x^4 = x^2 \Leftrightarrow x^2(x^2-1) = 0 \Rightarrow \begin{cases} x_1 = x_2 = 0 \\ x_3 = 1 \\ x_4 = -1 \end{cases}$

18. $x(x-a) = 3a \Leftrightarrow x^2 - ax - 3a = 0 \Rightarrow x = \frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 + 3a}$

$$\frac{a^2}{4} + 3a < 0 \Leftrightarrow a^2 + 12a = a(a+12) < 0 \Rightarrow -12 < a < 0$$

19. $x^3 + x + (2x+1)(x^2+1) = 0 \Leftrightarrow x(x^2+1) + (2x+1)(x^2+1) = 0 \Leftrightarrow$

$$(x^2+1)(3x+1) = 0 \Rightarrow \begin{cases} x_1 = i \\ x_2 = -i \\ x_3 = -1/3 \end{cases}$$

20. $z^2 + 2iz + p = 0 \Rightarrow z = -i \pm \sqrt{i^2 - p} = -i \pm \sqrt{-1-p} \Rightarrow p = -5$
$z_2 = -i - 2$

21. a)

$$\sqrt{x+6} = x \Leftrightarrow x + 6 = x^2 \Leftrightarrow x^2 - x - 6 = 0 \Leftrightarrow x = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 6} = \frac{1 \pm 5}{2}$$

$$\begin{cases} x_1 = 3 \\ x_2 = -2, \text{falsk rot} \end{cases}$$

b) $\sqrt{x+6} = 10x \Leftrightarrow x + 6 = 100x^2 \Leftrightarrow x^2 - 0.01x - 0.06 = 0$

$$x = \frac{1}{200} \pm \sqrt{\left(\frac{1}{200}\right)^2 + \frac{6}{100}} = \frac{1}{200} \pm \sqrt{\frac{1}{400} + \frac{6 \cdot 4 \cdot 100}{4 \cdot 100 \cdot 100}} = \frac{1 \pm 49}{200}$$

$$\begin{cases} x_1 = \frac{1}{4} \\ x_2 = -\frac{6}{25}, \text{falsk rot} \end{cases}$$

22. $x(x+12) = 189 \Leftrightarrow x^2 + 12x - 189 = 0 \Rightarrow x = -6 \pm \sqrt{36 - 189} = -6 \pm 15$

$$x = 9 \Rightarrow O = 2 \cdot (9 + 21) = 60 \text{ cm}$$

23. a) $200 - 7000h^2 = 100h \Leftrightarrow h^2 + \frac{1}{70}h - \frac{2}{70} = 0 \Rightarrow h = -\frac{1}{140} \pm \sqrt{\left(\frac{1}{140}\right)^2 + \frac{2}{70}} =$

$$= -\frac{1}{140} \pm \sqrt{\left(\frac{1}{140}\right)^2 + \frac{2 \cdot 2 \cdot 140}{70 \cdot 2 \cdot 140}} = \frac{-1 \pm 23.7}{140} = \begin{cases} h_1 = -0.176 \\ h_2 = 0.162 \end{cases}$$

b) $0.049h^2 + 6.5h + 100 = 0 \Rightarrow h^2 + \frac{6.5}{0.049}h + \frac{100}{0.049} = 0 \Rightarrow$

$$h = -\frac{6.5}{0.098} \pm \sqrt{\left(\frac{6.5}{0.098}\right)^2 - \frac{100}{0.049}} = \begin{cases} h_1 = -115 \\ h_2 = -17.8 \end{cases}$$

24. $\begin{cases} a \cdot b = 3847102 \\ a - b = 39 \end{cases} \Rightarrow a - \frac{3\,847\,102}{a} = 39 \Rightarrow a^2 - 39a - 3\,847\,102 = 0$

$$a = 19.5 \pm \sqrt{19.5^2 + 3\,847\,102} = \begin{cases} a_1 = 1981 \\ (a_2 = -1942) \end{cases}, b = 1942$$

Magnus är född 1942.

25. $x^2 + (x+10)^2 = 1.25^2(x+10)^2 \Rightarrow x^2 = (1.25^2 - 1)(x+10)^2 \Rightarrow$

$$x^2 = \left(\frac{25}{16} - 1\right)(x+10)^2 = \frac{9}{16}(x+10)^2 \Rightarrow x = \pm\frac{3}{4}(x+10) \Rightarrow$$

$$\Rightarrow \begin{cases} 4x = 3x + 30 \\ -4x = 3x + 30 \end{cases} \Rightarrow \begin{cases} x = 30 \\ -7x = 30 \text{ falsk rot} \end{cases} \quad O = 120 \text{ m}$$

Blandade uppgifter i kapitel 1

32. $(x-2)(x^8 - 81) = (x-2)(x^4 + 9)(x^4 - 9) = (x-2)(x^4 + 9)(x^2 + 3)(x^2 - 3) \Rightarrow$

$$x_1 x_2 x_3 = 2\left(-\sqrt{3}\right)\sqrt{3} = -6$$

33. Standard: $x^2 + \left(\frac{4}{3}\right)^2 x^2 = 28^2 \Rightarrow \frac{25}{9}x^2 = 28^2 \Rightarrow x = \sqrt{\frac{9 \cdot 28^2}{25}} \Rightarrow A = \frac{4}{3}\frac{9 \cdot 28^2}{25} \approx 376 \text{ tum}^2$

Bredbild: $y^2 + \left(\frac{16}{9}\right)^2 y^2 = 28^2 \Rightarrow \frac{337}{81}y^2 = 28^2 \Rightarrow y = \sqrt{\frac{81 \cdot 28^2}{337}} \Rightarrow A = \frac{16}{9}\frac{81 \cdot 28^2}{337} \approx 335 \text{ tum}^2$

34. a) $x \cdot 1.25x = x + 1.25x \Rightarrow x \cdot 1.25 = 2.25 \Rightarrow x = 1.8 \text{ och } 2.25$

b) $4x \cdot 5x = 2x \cdot 3x + 4046 \Rightarrow 14x^2 = 4046 \Rightarrow x = 17$
Talen är: 34, 51, 68, 85 dvs summan är 238.

35. $x(32 - x) + (x + 2)(30 - x) = 510$

$$32x - x^2 - x^2 + 28x + 60 = 510 \Rightarrow x^2 - 30x + 225 = 0$$

$$x = 15 \pm \sqrt{15^2 - 225} = 15 \text{ cm}$$

36.

$$(x+1)^3 = x^3+1 \Rightarrow x^3+3x^2+3x+1 = x^3+1 \Rightarrow 3x(x+1) = 0 \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -1 \end{cases}$$

37. $(x+a)(x-4) = a(x-2) \Rightarrow x^2-4x+ax-4a = ax-2a \Rightarrow$

$$x^2-4x-2a = 0 \Rightarrow x = 2 \pm \sqrt{4+2a} \Rightarrow a = -2 \text{ ger dubbelrot}$$

38. $(x+2+3y)^2-(x+3y)^2 = (x+2+3y+x+3y)\big(x+2+3y-(x+3y)\big) =$

$$= (2x+2+6y)(x+2+3y-x-3y) = 2(x+1+3y)2$$

Är jämnt delbart med 4 om *x* och *y* är hela tal.

40. $ax^2+25a = 3x \Rightarrow\ x^2-\frac{3}{a}3x+25 = 0 \Rightarrow x = \frac{3}{2a} \pm \sqrt{\frac{9}{4a^2}-25} \Rightarrow \frac{9}{4a^2}-25 < 0$
$\Rightarrow \frac{9}{4a^2} < 25 \Rightarrow \frac{9}{100} < a^2 \Rightarrow \frac{3}{10} < |a|$

42. $x^2-15ax+9 = 0 \Rightarrow x = 7.5a \pm \sqrt{(7.5a)^2-9} \Rightarrow (7.5a)^2-9 > 0 \Rightarrow$

$$(7.5a)^2 > 9 \Rightarrow a^2 > 0.16 \Rightarrow |a| > 0.4$$

43. $(x^2-2x)^{1/2} = \sqrt{x^2-2x} = \sqrt{-5} \Rightarrow x^2-2x+5 = 0 \Rightarrow x = 1 \pm \sqrt{1-5} \Rightarrow$

$$x = 1 \pm \sqrt{-4} = 1 \pm 2i \Rightarrow (1+2i)^2+(1-2i)^2 = 1+4i-4+1-4i-4 =$$

$$= 1-4+1-4 = -6$$

44. $x_1^2+x_2^2 = 45$ och $\frac{1}{x_1}+\frac{1}{x_2} = \frac{x_1+x_2}{x_1x_2} = \frac{1}{2} \Rightarrow 4(x_1+x_2) = 2x_1x_2 \Rightarrow$

$$x_1^2+x_2^2+2x_1x_2 = 45+4(x_1+x_2) \Rightarrow (x_1+x_2)^2 = 45+4(x_1+x_2)$$

$$\Rightarrow (-p)^2 = 45+4(-p) \Rightarrow p^2+4p-45 = 0 \Rightarrow p = -2 \pm \sqrt{4+45}$$

$$\Rightarrow p = -2 \pm 7 = \begin{cases} 5 \\ -9 \end{cases} \text{ och } -4p = \{\text{se uppgift 1352}\} = 2q \Rightarrow q = -2p = \begin{cases} -10 \\ 18 \end{cases}$$

$$x^2+5x-10 = 0 \text{ eller } x^2-9x+18 = 0$$

45. Kalle får 1 ± 3 med $q_{fel} \Rightarrow p_{rätt} = -2$
Pelle får -1 ∓ 2 med p_{fel}.
Använder vi Pelles svar för att hitta $q_{rätt}$ fås:

$$4 = \left(\frac{-2}{2}\right)^2 - q_{rätt} \Rightarrow q_{rätt} = -3 \text{ Svar: } x^2-2x-3 = 0$$

$$\Big(\big((m \cdot 100 + d) \cdot 2 + 5\big)10 + 23\Big)5 + \text{ålder} =$$
$$\big((m \cdot 100 + d) \cdot 2 + 5\big)50 + 115 + \text{ålder} =$$
$$(m \cdot 100 + d) \cdot 100 + 250 + 115 + \text{ålder} =$$
$$(m \cdot 100 + d) \cdot 100 + 365 + \text{ålder} =$$
$$m \cdot 100 \cdot 100 + d \cdot 100 + 365 + \text{ålder} =$$
$$m \cdot 10000 + d \cdot 100 + 365 + \text{ålder} =$$

Subtraheras 365 kommer åldern att ockupera de första två siffrorna till höger, dagen den tredje och eventuellt fjärde från höger, kvar den femte och eventuellt den sjätte till månaden.

VSV