

Valda uppgifter i kursboken Matematik M2c av Sjunnesson med flera utgiven på Liber, (2011).

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Kapitel 1

$$1113. \text{ a) } 1.4x - \frac{2}{5} \cdot (5 + 3.5x) = 1.4x - \left(2 + \frac{7}{5}x\right) = 1.4x - 2 - 1.4x = -2$$

$$\text{b) } 1.5(2x - y) - (x - 0.6y) \cdot 3 = 3x - 1.5y - (3x - 1.8y) = 0.3y$$

$$1114. \text{ a) } \frac{2}{3}(9 - 3x + 2y) - \left(\frac{y}{3} - \frac{x}{4} + 0.5\right) \cdot 4 = 6 - 2x + \frac{4}{3}y - \left(\frac{4}{3}y - x + 2\right) = 4 - x$$

$$\text{b) } 4 - x = 4 - (-5) = 4 + 5 = 9$$

$$1115. \text{ a) } C = A - B = 3 - 5x - (x + 3) = 3 - 5x - x - 3 = -6x$$

$$\text{b) } C = 2B - 3A = 2(x + 3) - 3(3 - 5x) = 2x + 6 - 9 + 15x = 17x - 3$$

$$\text{c) } C = 3B - Ax = 3(x + 3) - (3 - 5x)x = 3x + 9 - 3x + 5x^2 = 9 + 5x^2$$

$$1116. \text{ a) } \frac{x}{2} - \frac{3x}{2} + \frac{5x}{2} - \frac{7x}{2} + \frac{9x}{2} = \frac{x-3x+5x-7x+9x}{2} = \frac{5x}{2}$$

$$\text{b) } \frac{5x}{2} = \frac{5 \cdot 16}{2} = 40 \quad \text{c) } \frac{5x}{2} = \frac{5}{2} \cdot \left(-\frac{2}{15}\right) = -\frac{1}{3}$$

$$1117. \frac{7-2x}{3} - \frac{5-2x}{3} = \frac{7-2x-(5-2x)}{3} = \frac{7-2x-5+2x}{3} = \frac{2}{3}$$

$$1118. \text{ a) } \bar{x} = \frac{1}{4}(2x + 9 + 13 + x + 3(x - 8) + 2(1 + x)) = \frac{1}{4}8x = 2x$$

$$\text{b) } \bar{x} = 2 \cdot 9.5 = 19 \quad \text{c) } \bar{x} = -2 \cdot \frac{3}{8} = -\frac{3}{4}$$

$$1119. \text{ a) } \sqrt{3}(\sqrt{12} + \sqrt{3}) = (\sqrt{3}\sqrt{12} + \sqrt{3}\sqrt{3}) = \sqrt{3 \cdot 12} + 3 = \sqrt{36} + 3 = 9$$

$$\text{b) } \sqrt{2}(\sqrt{8} - \sqrt{18}) = (\sqrt{2}\sqrt{8} - \sqrt{2}\sqrt{18}) = \sqrt{16} - \sqrt{36} = 4 - 6 = -2$$

$$1120. 1 + x(2 - x) \cdot x - x^2(1 - x) = 1 + x^2(2 - x) - x^2(1 - x) = \\ = 1 + (2x^2 - x^3) - (x^2 - x^3) = 1 + x^2 \geq 1 > 0$$

1135.

$$2(x + 4) + x = 50 \Rightarrow 3x = 42 \Rightarrow x = 14 \text{ cm}$$

1136.

$$(400 + 4x) \cdot 3 + 2(350 + 3x) = 2800 \Rightarrow$$

$$1200 + 12x + 700 + 6x = 2800 \Rightarrow 18x = 900 \Rightarrow x = 50$$

1137. a)

$$\begin{cases} B = 5000 + E \\ F = 3 \cdot E \\ B + F + E = 20\,000 \end{cases} \Rightarrow 5000 + E + 3E + E = 20\,000 \Rightarrow \begin{cases} E = 3000 \\ B = 8000 \\ F = 9000 \end{cases}$$

$$\text{b) } \begin{cases} B = 5000 + E \\ F = 3 \cdot E \\ B + F + E = 40\,000 \end{cases} \Rightarrow 5000 + E + 3E + E = 40\,000 \Rightarrow \begin{cases} E = 7\,000 \\ B = 12\,000 \\ F = 21\,000 \end{cases}$$

$$1138. \text{ a) } 20 + \frac{x}{6} = x \Rightarrow 120 + x = 6x \Rightarrow x = \frac{120}{5} = 24$$

$$\text{b) } \frac{y}{3} + \frac{y}{2} = 15 \Rightarrow 6\frac{y}{3} + 6\frac{y}{2} = 6 \cdot 15 \Rightarrow 2y + 3y = 90 \Rightarrow y = \frac{90}{5} = 18$$

$$1139. \text{ a) } \frac{3p}{2} + \frac{9}{10} = \frac{7p}{5} \{\text{förläng med } 10\} \Rightarrow 5 \cdot 3p + 9 = 2 \cdot 7p \Rightarrow p = -9$$

$$\text{b) } \frac{1}{x} + \frac{2}{5} = \frac{3}{2x} \{\text{förläng med } 10x\} \Rightarrow 10 + 4x = 15 \Rightarrow x = \frac{5}{4}$$

1140. a)

$$9 - \frac{3(4x - 2)}{5} = \frac{2 + x}{10}$$

förläng med 10:

$$90 - 6(4x - 2) = 2 + x \Rightarrow 90 - 24x + 12 = 2 + x \Rightarrow 100 = 25x \Rightarrow x = 4$$

b)

$$\frac{6y - 2}{5} - \frac{4y - 2}{3} = 1 - \frac{8 + y}{6}$$

förläng med 30:

$$30 \cdot \frac{6y - 2}{5} - 30 \cdot \frac{4y - 2}{3} = 30 \cdot \left(1 - \frac{8 + y}{6}\right)$$

$$6 \cdot (6y - 2) - 10 \cdot (4y - 2) = 30 - 5(8 + y)$$

$$36y - 12 - (40y - 20) = 30 - (40 + 5y)$$

$$36y - 12 - 40y + 20 = 30 - 40 - 5y$$

$$8 - 4y = -10 - 5y \Rightarrow y = -18$$

1141.

$$x - 12 = \frac{x}{3} \Rightarrow 3x - 36 = x \Rightarrow 2x = 36 \Rightarrow x = 18$$

$$1142. \begin{cases} J = 2 \cdot E \\ M = J + 4 \\ E + J + M = 64 \end{cases} \Rightarrow \begin{cases} J = 2 \cdot E \\ M = 2E + 4 \\ E + 2E + 2E + 4 = 64 \end{cases} \Rightarrow 5E = 60 \Rightarrow M = 28 \text{ år}$$

$$1143. \frac{x+2}{3x+6} = \frac{1}{3} \Rightarrow \frac{x+2}{3(x+2)} = \frac{1}{3} \text{ gäller för alla } x \text{ utom } x = -2.$$

$$1144. \text{ a) } \frac{1}{6} = \frac{2}{x-1} \Rightarrow x - 1 = 12 \Rightarrow x = 13$$

$$\text{ b) } \frac{3}{x-2} = \frac{2}{x+1} \Rightarrow 3(x+1) = 2(x-2) \Rightarrow x = -7$$

$$1145. \frac{x}{25+x} = 0.02 \Rightarrow x = 0.5 + 0.02x \Rightarrow 0.98x = 0.5 \Rightarrow x = \frac{0.5}{0.98} \approx 5.1 \text{ dl}$$

$$1146. \frac{1}{6} = \frac{2}{3-x^2} \Rightarrow 3 - x^2 = 12 \Rightarrow x^2 = -9 \text{ dvs inga reella rötter.}$$

1211.

$$\begin{aligned} 3x(2x-5) - (x+8)(x-1) + 2(2.5x-1)(4-x) &= \\ = 6x^2 - 15x - (x^2 - x + 8x - 8) + (5x-2)(4-x) &= \\ = 6x^2 - 15x - x^2 + x - 8x + 8 + 20x - 5x^2 - 8 + 2x &= \\ = -15x + x - 8x + 20x + 2x &= 0 \end{aligned}$$

1213.

$$\begin{aligned} 3 - (x-7)(2x+6) &= 3 - (2x^2 + 6x - 14x - 42) = \\ = 3 - 2x^2 - 6x + 14x + 42 &= 45 + 8x - 2x^2 \end{aligned}$$

1214.

$$\begin{aligned} (x+y+2)(3+x) - (x-y)(-3-x) &= (x+y+2)(3+x) + (x-y)(3+x) = \\ = (x+y+2+(x-y))(3+x) &= (2+2x)(3+x) = 6 + 8x + 2x^2 \end{aligned}$$

1215. Kalla sidorna x , y , u och z analogt med problem 1212. Då gäller:

$$\begin{cases} xy = 117 \\ uy = 135 \\ xz = 156 \end{cases} \Rightarrow uz = \frac{135}{y} \cdot \frac{156}{x} = \frac{135 \cdot 156}{117} = 180 \text{ m}^2$$

$$1225. (a-b) \cdot (a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$$

$$1226. (a-b)^2 = a^2 - 2ab + b^2 = b^2 - 2ab + a^2 = (b-a)^2$$

$$1227. \text{ a) } (\sqrt{2} + \sqrt{8})^2 = (\sqrt{2})^2 + 2\sqrt{2}\sqrt{8} + (\sqrt{8})^2 = 2 + 2\sqrt{16} + 8 = 18$$

$$\text{ b) } (\sqrt{12} - \sqrt{3})^2 = (\sqrt{12})^2 - 2\sqrt{12}\sqrt{3} + (\sqrt{3})^2 = 12 - 2\sqrt{36} + 3 = 3$$

1228. a)

$$(x+h)^3 - (x-h)^3 = x^3 + 3x^2h + 3xh^2 + h^3 - (x^3 - 3x^2h + 3xh^2 - h^3) =$$

$$= x^3 + 3x^2h + 3xh^2 + h^3 - x^3 + 3x^2h - 3xh^2 + h^3 = 6x^2h + 2h^3$$

b)

$$(x+2)^3 - (x+2)^2 - (x-2) =$$

$$= x^3 + 3x^2 \cdot 2 + 3x \cdot 2^2 + 2^3 - (x^2 + 4x + 4) - (x-2) =$$

$$= x^3 + 6x^2 + 12x + 8 - x^2 - 4x - 4 - x + 2 =$$

$$= x^3 + 5x^2 + 7x + 6$$

$$1229 \text{ a) } (\sqrt{2x} + \sqrt{4.5x})^2 = 2x + 2\sqrt{2x}\sqrt{4.5x} + 4.5x = 6.5x + 2 \cdot 3x = 12.5x$$

$$\text{b) } (\sqrt{20x} - x\sqrt{5x})^2 = 20x - 2\sqrt{20x} \cdot x\sqrt{5x} + x^2 5x = 20x - 2x \cdot 10x + x^2 5x =$$

$$= 20x - 20x^2 + 5x^3$$

$$\text{c) } (5^{3x} - 5^{0.5x})^2 = 5^{6x} - 2 \cdot 5^{3x} 5^{0.5x} + 5^x = 5^{6x} - 2 \cdot 5^{3.5x} + 5^x$$

$$\text{d) } \left(\frac{a+1}{2}\right)^2 - \left(\frac{a-1}{2}\right)^2 = \frac{a^2+2a+1}{4} - \frac{a^2-2a+1}{4} = \frac{a^2+2a+1-a^2+2a-1}{4} = a$$

1230. Hela den stora kvadraten = 4 gula plus den röda. Eller matematiskt uttryckt:

$$c^2 = 4 \frac{ab}{2} + (a-b)^2 = 2ab + a^2 - 2ab + b^2 = a^2 + b^2$$

1243.

$$\begin{cases} a-b=5 \\ a^2-b^2=195 \end{cases} \Rightarrow \begin{cases} a-b=5 \\ (a+b)(a-b)=195 \end{cases} \Rightarrow \begin{cases} a-b=5 \\ (a+b)5=195 \end{cases} \Rightarrow \begin{cases} a-b=5 \\ a+b=39 \end{cases}$$

$$\Rightarrow \begin{cases} a=22 \\ b=17 \end{cases}$$

$$1244. \text{ a) } 18^2 = 20 \cdot 16 + 4 = 320 + 4 = 324, \quad 14^2 = 10 \cdot 18 + 16 = 196$$

$$39^2 = 40 \cdot 38 + 1 = 1520 + 1 = 1521, \quad 97^2 = 100 \cdot 94 + 9 = 9409$$

$$\text{b) } a^2 = (a-b)(a+b) + b^2 = a^2 - b^2 + b^2 = a^2 \text{ VSV}$$

1254.

$$(x+3)^2 + 8(3-x)^2 = 43 - (5+3x)(5-3x)$$

$$(x^2 + 6x + 9) + 8(9 - 6x + x^2) = 43 - (25 - 9x^2)$$

$$x^2 + 6x + 9 + 72 - 48x + 8x^2 = 43 - 25 + 9x^2$$

$$9x^2 + 81 - 42x = 18 + 9x^2 \Rightarrow 63 = 42x \Rightarrow x = \frac{63}{42} = \frac{3}{2} = 1.5$$

$$1255. (3x + 4)^2 - (4 - 3x)^2 = (2x + 3)^2 - (3 - 2x)^2$$

$$(9x^2 + 24x + 16) - (16 - 24x + 9x^2) = (4x^2 + 12x + 9) - (4x^2 - 12x + 9)$$

$$48x = 24x \Rightarrow x = 0$$

$$1256. x^2 + x + 19 = (x + 3)^2 = x^2 + 6x + 9 \Rightarrow 5x = 10 \Rightarrow x = 2$$

$$1257. 5^2 + x^2 = (10 - x)^2 = 100 - 20x + x^2 \Rightarrow 20x = 75 \Rightarrow x = \frac{15}{4} = 3.75 \text{ cm}$$

$$1258. a) 14^2 + (35 - x)^2 = (35 + x)^2 \Rightarrow 196 - 70x = 70x \Rightarrow x = \frac{196}{140} = \frac{14 \cdot 14}{10 \cdot 14} = 1.4 \text{ cm}$$

Svar: Konens höjd är 33.6 cm

$$b) V_{kon} = \frac{bh}{3} = \frac{3.36}{3} \pi 1.4^2 \approx 7 \text{ liter}$$

1312. Fall 1: 15 är en katet och $4x$ är hypotenusan.

$$15^2 + 4x^2 = 16x^2 \Rightarrow 12x^2 = 225 \Rightarrow x = \sqrt{\frac{225}{12}} \approx 4.3 \text{ cm}$$

Svar: $2x = 8.7 \text{ cm}$

Fall 2: $4x$ är en katet och 15 är hypotenusan.

$$15^2 = 4x^2 + 16x^2 \Rightarrow 20x^2 = 225 \Rightarrow x = \sqrt{\frac{225}{20}} \approx 3.4 \text{ cm}$$

Svar: $2x = 6.7 \text{ cm}$

$$1313. a) \frac{x}{3} = \frac{27}{x} \Rightarrow x^2 = 81 \Rightarrow x = \pm 9$$

$$b) 3x = (\sqrt{x} + 3)(\sqrt{x} - 3) + 10 = x - 9 + 10 = x + 1 \Rightarrow 2x = 1 \Rightarrow x = 0.5$$

$$1314. \frac{3x-2}{x-1} = \frac{x-1}{x} \Rightarrow (3x-2)x = (x-1)^2 \Rightarrow$$

$$3x^2 - 2x = x^2 - 2x + 1 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm \frac{1}{\sqrt{2}} = \pm \sqrt{0.5}$$

$$1315. (x + a)^2 = (2a - 3x)^2 \Rightarrow \begin{cases} x_1 + a = 2a - 3x_1 \\ x_2 + a = -2a + 3x_2 \end{cases} \Rightarrow \begin{cases} 4x_1 = a \\ 2x_2 = 3a \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = \frac{a}{4} = 0.25a \\ x_2 = \frac{3a}{2} = 1.5a \end{cases}$$

$$1316. (6 - a)^2 = 16 \Rightarrow 6 - a = \pm 4 \Rightarrow a = 6 \mp 4 \Rightarrow \begin{cases} a_1 = 2 \\ a_2 = 10 \end{cases} \Rightarrow$$

$$\begin{cases} (x-2)^2 = 16 \\ (x-10)^2 = 16 \end{cases} \Rightarrow \begin{cases} x_1 = 6, x_2 = -2 \\ x_3 = 14, x_4 = 6 \end{cases}$$

1322.

$$(x+6)^2 + x^2 = 20^2 \Rightarrow x^2 + 12x + 36 + x^2 = 400 \Rightarrow 2x^2 + 12x - 364 = 0 \Rightarrow$$

$$x^2 + 6x - 182 = 0 \Rightarrow x = -3 \pm \sqrt{9 + 182} \approx -3 \pm 13.8 = 10.8 \text{ cm}$$

$$A_{\Delta} = \frac{10.8(10.8 + 6)}{2} \approx 91 \text{ cm}^2$$

$$1323. y^4 - 10y^2 + 9 = 0 \Rightarrow y^2 = 5 \pm \sqrt{25 - 9} = 5 \pm 4 = \begin{cases} y_1^2 = 9 \\ y_2^2 = 1 \end{cases} \Rightarrow \begin{cases} y_1 = \pm 3 \\ y_2 = \pm 1 \end{cases}$$

$$1338. (x+1)(x-3) = 0 \Rightarrow x^2 - 3x + x - 3 = 0 \Rightarrow x^2 - 2x - 3 = 0$$

$$1339. 1.2 = 0.0001x^2 - 0.016x + 1.34 \Rightarrow 0.0001x^2 - 0.016x + 0.14 = 0 \Rightarrow$$

$$x^2 - 160x + 1400 = 0 \Rightarrow x = 80 \pm \sqrt{80^2 - 1400} \approx 150 \text{ km/h}$$

$$1340. \text{ a) } x^4 - 13x^2 + 36 = 0 \Rightarrow x^2 = 6.5 \pm \sqrt{6.5^2 - 36} = 6.5 \pm 2.5 = \begin{cases} 9 \\ 4 \end{cases} \Rightarrow$$

$$x^2 = \begin{cases} 9 \\ 4 \end{cases} \Rightarrow x = \begin{cases} \pm 3 \\ \pm 2 \end{cases}$$

$$\text{ b) } x - 5\sqrt{x} + 4 = 0 \Rightarrow \sqrt{x} = 2.5 \pm \sqrt{2.5^2 - 4} = \begin{cases} 4 \\ 1 \end{cases} \Rightarrow x = \begin{cases} 16 \\ 1 \end{cases}$$

$$1341. \text{ a) } ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \Rightarrow p = \frac{b}{a} \text{ och } q = \frac{c}{a}$$

$$\text{ b) } x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}} = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ VSB}$$

$$1352. \text{ a) } a = 5 \cdot 6, b = -11 \Rightarrow x^2 - 11x + 30 = 0$$

$$\text{ b) } \begin{cases} r_1 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q} \\ r_2 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q} \end{cases} \Rightarrow \begin{cases} r_1 + r_2 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q} - \frac{p}{2} - \sqrt{\frac{p^2}{4} - q} \\ r_1 r_2 = \left(-\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}\right) \left(-\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}\right) \end{cases}$$

$$\begin{cases} r_1 + r_2 = -p \\ r_1 r_2 = \frac{p^2}{4} - \left(\frac{p^2}{4} - q\right) = q \end{cases}$$

1362. a)

n	1	2	3	4	5	6	7	8	9	10	11	12
i^n	i	-1	$-i$	1	i	-1	$-i$	1	i	-1	$-i$	1

b) $i^n = 1$ när n är delbart med 4.

c) $i^{87} + i^{4001} = i^{84+3} + i^{4000+1} = i^3 + i^1 = -i + i = 0$

1363.

$$2z^2 + 2(4 - 2i)z - 16i = 0 \Rightarrow z^2 + (4 - 2i)z - 8i = 0$$

$$z = -\frac{4 - 2i}{2} \pm \sqrt{\left(\frac{4 - 2i}{2}\right)^2 + 8i} = i - 2 \pm \sqrt{\frac{16 - 16i - 4}{4} + 8i} =$$
$$= i - 2 \pm \sqrt{3 - 4i + 8i} = i - 2 \pm \sqrt{3 + 4i} = i - 2 \pm (2 + i) = \begin{cases} 2i \\ -4 \end{cases}$$

1364. $z_1 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}$, $z_2 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}$, $z_1 + z_2 = -p = 12$,
 $z_1 = a + bi$ och $z_2 = a - bi \Rightarrow z_1 + z_2 = 2a = 12 \Rightarrow a = 6$

Då rötterna är komplexa måste:

$$\frac{p^2}{4} - q < 0 \Rightarrow \frac{12^2}{4} - q < 0 \Rightarrow 36 - q < 0 \Rightarrow q > 36$$

1370. a) $\sqrt{3 - x} = -x - 9 \Rightarrow 3 - x = x^2 + 18x + 81$

$$\Rightarrow x^2 + 19x + 78 = 0 \Rightarrow x = -9.5 \pm \sqrt{9.5^2 - 78} = -9.5 \pm 3.5 = \begin{cases} x_1 = -6 \\ x_2 = -13 \end{cases}$$

Test: $\begin{cases} \sqrt{3 - (-6)} = -(-6) - 9 \\ \sqrt{3 - (-13)} = -(-13) - 9 \end{cases} \Rightarrow \begin{cases} \sqrt{9} = -3 \text{ NEJ, falsk rot!} \\ \sqrt{16} = 4 \text{ Stämmer!} \end{cases}$

b) $\sqrt{x^2 + \sqrt{20x + 70}} = x + 2 \Rightarrow x^2 + \sqrt{20x + 70} = x^2 + 4x + 4 \Rightarrow$

$$\Rightarrow \sqrt{20x + 70} = 4x + 4 \Rightarrow 20x + 70 = 16x^2 + 32x + 16 \Rightarrow$$

$$\Rightarrow 16x^2 + 12x - 54 = 0 \Rightarrow x^2 + \frac{3}{4}x - \frac{27}{8} = 0 \Rightarrow x = -\frac{3}{8} \pm \sqrt{\left(\frac{3}{8}\right)^2 + \frac{27}{8}}$$

$$\Rightarrow x = -\frac{3}{8} \pm \sqrt{\frac{225}{64}} = \frac{-3 \pm 15}{8} = \begin{cases} x_1 = \frac{3}{2} \text{ Riktig rot!} \\ x_2 = -\frac{9}{4} \text{ falsk rot.} \end{cases}$$

1371. A. $\sqrt{2x + 5} = x + 1 \Rightarrow 2x + 5 = x^2 + 2x + 1 \Rightarrow x = \pm 2$ men -2 är en falsk rot.

B. $\sqrt{2x + 5} = -(x + 1) \Rightarrow$ Bara $x = -2$ är lösning.

C. $2x + 5 = (x + 1)^2 \Rightarrow x = \pm 2$ båda är riktiga rötter.

1381.

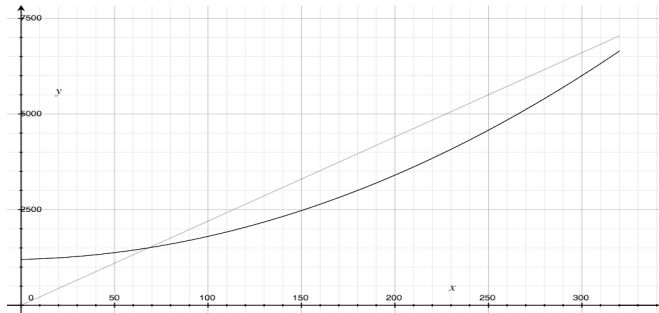
$$a) 6000 = 1200 + x + 0.05x^2 \Rightarrow x^2 + 20x - 96000 = 0 \Rightarrow$$

$$x = -10 \pm \sqrt{100 + 96000} = 300 \text{ st (den negativa roten stryks)}$$

$$b) 22x = 1200 + x + 0.05x^2 \Rightarrow x^2 - 420x + 24000 = 0$$

x

$$= 210 \pm \sqrt{210^2 - 24000} = \begin{cases} 68 \text{ flaskor} \\ 352 \end{cases} \text{ (utanför definitionsmängden)}$$



1382.

$$\begin{cases} a + b = 51 \\ ab = 144 \end{cases} \Rightarrow a + \frac{144}{a} = 51 \Rightarrow a^2 - 51a + 144 = 0 \Rightarrow a = 25.5 \pm \sqrt{25.5^2 - 144}$$

$$a = 25.5 \pm 22.5 = \begin{cases} 48 \\ 3 \end{cases} \Rightarrow \begin{cases} a_1 = 48 \\ b_1 = 3 \end{cases} \text{ eller } \begin{cases} a_2 = 3 \\ b_2 = 48 \end{cases}$$

$$1383. h = r + 16, 105 = 2\pi r^2 + 2\pi r(r + 16) = 2\pi r^2 + 2\pi r^2 + 2\pi r16 \Rightarrow$$

$$4\pi r^2 + 2\pi r16 - 105 = 0 \Rightarrow r^2 + \frac{2\pi 16}{4\pi} r - \frac{105}{4\pi} = 0 \Rightarrow r^2 + 8r - \frac{105}{4\pi} = 0 \Rightarrow$$

$$r = -4 \pm \sqrt{16 + \frac{105}{4\pi}} = 9.4 \text{ mm}$$

1384. Den lilla delen, $1-x$, förhåller sig till den stora som den stora till det hela.

$$\frac{1-x}{x} = \frac{x}{1} \Rightarrow 1-x = x^2 \Rightarrow x^2 + x - 1 = 0 \Rightarrow x = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 1}$$

$$\Rightarrow x = -\frac{1}{2} \pm \sqrt{\frac{1+4}{4}} = \{\text{ignorera den negativa lösningen}\} = \frac{\sqrt{5}-1}{2}$$

$$1412. a) x^3 - 81x = x(x^2 - 81) = x(x+9)(x-9)$$

$$b) 4(x+2)(x+7) - 8(x+2)(2x+3) = 4(x+2)((x+7) - 2(2x+3)) =$$

$$= 4(x+2)(x+7-4x-6) = 4(x+2)(1-3x)$$

$$c) (x + 3)^2 - 9y^2 = (x + 3 - 3y)(x + 3 + 3y)$$

$$d) b^{3n} - b^n = b^n(b^{2n} - 1) = b^n(b^n + 1)(b^n - 1)$$

$$1424. x^2 + a = 0 \Rightarrow (x + 7)(x - 7) = x^2 - 49 \Rightarrow a = -49$$

$$\begin{aligned} 1425. x^{80} - 1 &= (x^{40} + 1)(x^{40} - 1) = (x^{40} + 1)(x^{20} + 1)(x^{20} - 1) = \\ &= (x^{40} + 1)(x^{20} + 1)(x^{20} - 1) = (x^{40} + 1)(x^{20} + 1)(x^{10} + 1)(x^{10} - 1) = \\ &= (x^{40} + 1)(x^{20} + 1)(x^{10} + 1)(x^5 + 1)(x^5 - 1) \end{aligned}$$

$$\begin{aligned} 1439. z^5 - za^4 &= z(z^4 - a^4) = z(z^2 + a^2)(z^2 - a^2) = z(z^2 + a^2)(z + a)(z - a) \\ &= z(z^2 + 49)(z + 7)(z - 7) \Rightarrow z = 0, z = \pm 7 \text{ och } z = \pm 7i \end{aligned}$$

$$1440. f(3) = 3a(3 + a) = 12 \Rightarrow a^2 + 3a - 4 = 0 \Rightarrow a = -1.5 \pm \sqrt{2.25 + 4} = \begin{cases} -4 \\ 1 \end{cases}$$

$$f(x) = \begin{cases} -4x(x - 4) = 0 \Rightarrow x_1 = 0, x_2 = 4 \\ x(x + 1) = 0 \Rightarrow x_3 = 0, x_4 = -1 \end{cases}$$

Blandade uppgifter i kapitel 1

$$\begin{aligned} 32. (x - 2)(x^8 - 81) &= (x - 2)(x^4 + 9)(x^4 - 9) = (x - 2)(x^4 + 9)(x^2 + 3)(x^2 - 3) \Rightarrow \\ x_1 x_2 x_3 &= 2(-\sqrt{3})\sqrt{3} = -6 \end{aligned}$$

$$33. \text{Standard: } x^2 + \left(\frac{4}{3}\right)^2 x^2 = 28^2 \Rightarrow \frac{25}{9} x^2 = 28^2 \Rightarrow x = \sqrt{\frac{9 \cdot 28^2}{25}} \Rightarrow A = \frac{4}{3} \frac{9 \cdot 28^2}{25} \approx 376 \text{ tum}^2$$

$$\text{Bredbild: } y^2 + \left(\frac{16}{9}\right)^2 y^2 = 28^2 \Rightarrow \frac{337}{81} y^2 = 28^2 \Rightarrow y = \sqrt{\frac{81 \cdot 28^2}{337}} \Rightarrow A = \frac{16}{9} \frac{81 \cdot 28^2}{337} \approx 335 \text{ tum}^2$$

$$34. a) x \cdot 1.25x = x + 1.25x \Rightarrow x \cdot 1.25 = 2.25 \Rightarrow x = 1.8 \text{ och } 2.25$$

$$b) 4x \cdot 5x = 2x \cdot 3x + 4046 \Rightarrow 14x^2 = 4046 \Rightarrow x = 17$$

Talen är: 34, 51, 68, 85 dvs summan är 238.

$$35. x(32 - x) + (x + 2)(30 - x) = 510$$

$$32x - x^2 - x^2 + 28x + 60 = 510 \Rightarrow x^2 - 30x + 225 = 0$$

$$x = 15 \pm \sqrt{15^2 - 225} = 15 \text{ cm}$$

36.

$$(x+1)^3 = x^3 + 1 \Rightarrow x^3 + 3x^2 + 3x + 1 = x^3 + 1 \Rightarrow 3x(x+1) = 0 \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = -1 \end{cases}$$

37. $(x+a)(x-4) = a(x-2) \Rightarrow x^2 - 4x + ax - 4a = ax - 2a \Rightarrow$

$$x^2 - 4x - 2a = 0 \Rightarrow x = 2 \pm \sqrt{4 + 2a} \Rightarrow a = -2 \text{ ger dubbelrot}$$

38. $(x+2+3y)^2 - (x+3y)^2 = (x+2+3y+x+3y)(x+2+3y-(x+3y)) =$
 $= (2x+2+6y)(x+2+3y-x-3y) = 2(x+1+3y)2$

Är jämnt delbart med 4 om x och y är hela tal.

40. $ax^2 + 25a = 3x \Rightarrow x^2 - \frac{3}{a}3x + 25 = 0 \Rightarrow x = \frac{3}{2a} \pm \sqrt{\frac{9}{4a^2} - 25} \Rightarrow \frac{9}{4a^2} - 25 < 0$
 $\Rightarrow \frac{9}{4a^2} < 25 \Rightarrow \frac{9}{100} < a^2 \Rightarrow \frac{3}{10} < |a|$

42. $x^2 - 15ax + 9 = 0 \Rightarrow x = 7.5a \pm \sqrt{(7.5a)^2 - 9} \Rightarrow (7.5a)^2 - 9 > 0 \Rightarrow$
 $(7.5a)^2 > 9 \Rightarrow a^2 > 0.16 \Rightarrow |a| > 0.4$

43. $(x^2 - 2x)^{1/2} = \sqrt{x^2 - 2x} = \sqrt{-5} \Rightarrow x^2 - 2x + 5 = 0 \Rightarrow x = 1 \pm \sqrt{1 - 5} \Rightarrow$
 $x = 1 \pm \sqrt{-4} = 1 \pm 2i \Rightarrow (1 + 2i)^2 + (1 - 2i)^2 = 1 + 4i - 4 + 1 - 4i - 4 =$
 $= 1 - 4 + 1 - 4 = -6$

44. $x_1^2 + x_2^2 = 45$ och $\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1+x_2}{x_1x_2} = \frac{1}{2} \Rightarrow 4(x_1 + x_2) = 2x_1x_2 \Rightarrow$

$$x_1^2 + x_2^2 + 2x_1x_2 = 45 + 4(x_1 + x_2) \Rightarrow (x_1 + x_2)^2 = 45 + 4(x_1 + x_2)$$

$$\Rightarrow (-p)^2 = 45 + 4(-p) \Rightarrow p^2 + 4p - 45 = 0 \Rightarrow p = -2 \pm \sqrt{4 + 45}$$

$$\Rightarrow p = -2 \pm 7 = \begin{cases} 5 \\ -9 \end{cases} \text{ och } -4p = \{\text{se uppgift 1352}\} = 2q \Rightarrow q = -2p = \begin{cases} -10 \\ 18 \end{cases}$$

$$x^2 + 5x - 10 = 0 \text{ eller } x^2 - 9x + 18 = 0$$

45. Kalle får 1 ± 3 med $q_{fel} \Rightarrow p_{rätt} = -2$

Pelle får -1 ∓ 2 med p_{fel} .

Använder vi Pelles svar för att hitta $q_{rätt}$ fås:

$$4 = \left(\frac{-2}{2}\right)^2 - q_{rätt} \Rightarrow q_{rätt} = -3 \text{ Svar: } x^2 - 2x - 3 = 0$$