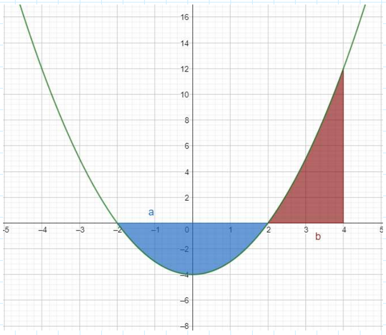
Kunskapsmatrisen: e7Z9GU

Från <https://www.kunskapsmatrisen.se/verktyg/skapaprov-oversikt.php>



https://ggbm.at/vshcfavh

$f(x) = x^2 - 4$ $\Rightarrow F(x) = \frac{x^3}{3} - 4x$

2 sätt

① $\int_{-2}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx$

② $\int_{-2}^{4} f(x)\,dx$

① $\int_{-2}^{2} f(x)\,dx = \left[\frac{x^3}{3} - 4x\right]_{-2}^{2} = \left[\frac{8}{3} - 4\cdot 2\right] - \left[\frac{-8}{3} - 4(-2)\right]$

$= \frac{8}{3} - 8 + \frac{8}{3} - 8 = \frac{16}{3} - 16 = \frac{16}{3} - \frac{48}{3} = -\frac{32}{3} \Rightarrow 32$ a.e

area kan inte vara negativ, men integral kan.

$\int_{2}^{4} f(x)\,dx = \left[\frac{x^3}{3} - 4x\right]_{2}^{4} = \left[\frac{4^3}{3} - 4\cdot 4\right] - \left[\frac{2^3}{3} - 4\cdot 2\right] =$

$\frac{64}{3} - 16 - \frac{8}{3} + 8 = \frac{56}{3} - 8 = \frac{56}{3} - \frac{24}{3} = \frac{32}{3}$ a.e

Svar: arean är $\frac{32}{3} + \frac{32}{3} = \frac{64}{3}$ a.e

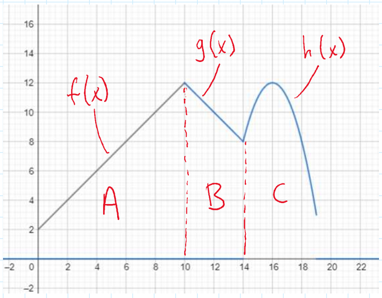
② $\int_{-2}^{4} f(x)\,dx = \left[\frac{x^3}{3} - 4x\right]_{-2}^{4} = \frac{4^3}{3} - 4\cdot 4 - \left[\frac{(-2)^3}{3} - 4(-2)\right] = \frac{64}{3} - 16 - \left[\frac{-8}{3} + 8\right]$

$= \frac{64}{3} - 16 + \frac{8}{3} - 8 = \frac{72}{3} - 24 = 24 - 24 = 0$

varför?

därför integralerna tar ut varandra.

https://www.geogebra.org/graphing/jseuwwcg



För att räkna ut arean under grafen delar jag in den i 3 områden: A, B, C

$f(x) = x + 2$ $\Rightarrow F(x) = \frac{x^2}{2} + 2x$

$g(x) = -x + 22$ $\Rightarrow G(x) = -\frac{x^2}{2} + 22x$

$h(x) = -x^2 + 32x - 244$ $\Rightarrow H(x) = -\frac{x^3}{3} + 16x^2 - 244x$

Räknar areorna A, B, C för sig och adderar sedan

A: $\int_0^{10} f(x)\,dx = \left[\frac{x^2}{2} + 2x\right]_0^{10} = \frac{100}{2} + 20 - \left[\frac{0}{2} + 0\right] = 70$ a.e

B: $\int_{10}^{14} g(x)\,dx = \left[-\frac{x^2}{2} + 22x\right]_{10}^{14} = -\frac{14^2}{2} + 22\cdot 14 - \left[-\frac{10^2}{2} + 22\cdot 10\right] = -\frac{14\cdot 14}{2} + 22\cdot 14 + 50 - 220$

$= -14\cdot 7 + 22\cdot 14 - 170 = 15\cdot 14 - 170 = 210 - 170 = 40$ a.e

C: $\int_{14}^{19} h(x)\,dx = \left[H(x)\right]_{14}^{19} = \left[-\frac{x^3}{3} + 16x^2 - 244x\right]_{14}^{19} = \left[\frac{19^3}{3} + 16\cdot 19^2 - 244\cdot 19\right] - \left[\frac{14^3}{3} + 16\cdot 14^2 - 244\cdot 14\right]$

$= \frac{19^3}{3} + 16\cdot 19^2 \underbrace{- 244\cdot 19 - \frac{14^3}{3} - 16\cdot 14^2 + 244\cdot 14}_{-5\cdot 244} = \frac{19^3 - 14^3}{3} + 16\cdot 19^2 - 16\cdot 14^2 - 5\cdot 244 = 48.33$ a.e

$A + B + C = 70 + 40 + 48 = 158.33$ a.e

https://www.geogebra.org/graphing/c4grwuec

Delar in arean i 2 områden A, B

$g(x) = -x^2 + 6x$ (grå) $\Rightarrow G(x) = -\frac{x^3}{3} + 3x^2$

$h(x) = -x^2 + 14x - 40$ (blå) $\Rightarrow H(x) = -\frac{x^3}{3} + 7x^2 - 40x$

$\Rightarrow H(x) = -\frac{x^3}{3} + 7x^2 - 40x$

$$A: \int_0^5 g(x)\,dx = \Big[G(x)\Big]_0^5 = \left[-\frac{x^3}{3} + 3x^2\right]_0^5 = \underbrace{\left[-\frac{5^3}{3} + 3\cdot 5^2\right]}_{= -\frac{125}{3} + 3\cdot 25} - \underbrace{\left[-\frac{0^3}{3} + 3\cdot 0^2\right]}_{=0}$$

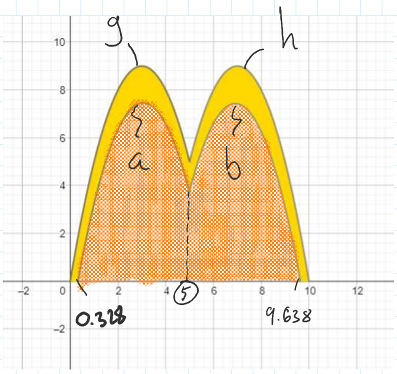
$$= -\frac{125}{3} + 75 = 33.3 \text{ a.e}$$

$$B: \int_5^{10} h(x)\,dx = \Big[H(x)\Big]_5^{10} = \left[-\frac{x^3}{3} + 7x^2 - 40x\right]_5^{10} = \left[-\frac{1000}{3} + 7\cdot 100 - 400\right] - \left[-\frac{125}{3} + 7\cdot 25 - 200\right] = -\frac{1000}{3} + \overbrace{700 - 400}^{300} + \frac{125}{3} \overbrace{- 175 + 200}^{+25}$$

$$= \frac{-1000 + 125}{3} + \underset{\frac{3\cdot 325}{3} = \frac{975}{3}}{325} = \frac{-875 + 975}{3} = \frac{+100}{3} = 33.33 \text{ a.e.}$$

Total area: $A + B = 33.33 + 33.33 = 66.66$ a.e.

https://www.geogebra.org/graphing/c4grwuec



Arean i det "gula" området är lika med arean för hela området minus det orangea området.
Hela arean fick vi från förra uppg.
$= 66.66$ a.e.

$g(x) = -x^2 + 6x$

$h(x) = -x^2 + 14x - 40$

$a(x) = -x^2 + 6.114x - 1.891$ $\Rightarrow A(x) = -\frac{x^3}{3} + \frac{6.114x^2}{2} - 1.891x$

$b(x) = -x^2 + 13.828x - 40.38$ $\Rightarrow B(x) = -\frac{x^3}{3} + \frac{13.828x^2}{2} - 40.38x$

Area för det orangea området fås av:

$$\underbrace{\int_{1.2}^{5} a(x)\,dx}_{(1)} + \underbrace{\int_{5}^{9.638} b(x)\,dx}_{(2)}$$

$$(1) \int_{0.328}^{5} a(x)\,dx = \Big[A(x)\Big]_{0.328}^{5} = \left[-\frac{x^3}{3} + 3.114x^2 - 1.891x\right]^5 =$$

(1) $\int_{0.328}^{5} a(x)\,dx = \left[A(x)\right]_{0.328}^{5} = \left[\frac{-x^3}{3} + 3.114x^2 - 1.891x\right]_{0.328}^{5} =$

$$\left[\underbrace{-\frac{125}{3} + 3.114 \cdot 25 - 1.891 \cdot 5}_{=26.73}\right] - \left[\underbrace{-\frac{0.328^3}{3} + 3.114 \cdot 0.328^2 - 1.891 \cdot 0.328}_{=-0.297}\right] = 26.433$$

(2) $\int_{5}^{9.638} b(x)\,dx = \left[B(x)\right]_{5}^{9.638} = \left[\frac{-x^3}{3} + \frac{13.828}{2}x^2 - 40.38 \cdot x\right]_{5}^{9.638} =$

$$\left[\underbrace{-\frac{9.638^3}{3} + \frac{13.828}{2} \cdot 9.638^2 - 40.38 \cdot 9.638}_{=-45.362}\right] - \left[\underbrace{-\frac{125}{3} + \frac{13.828}{2} \cdot 25 - 40.38 \cdot 5}_{=-70.7}\right] \approx 25$$

Area orange: $26.433 + 25 \approx 51$ a.e

Svar: Area $= 66.66 - 51 = 15.66$ a.e

"M"